LECTURE 2.
Status of low-energy quadrupole "beta" vibrations in deformed nuclei

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W.D. Kulp et al., Phys. Rev. Lett. 91 102501 2003
W.D. Kulp et al., Phys. Rev. C 69 064309 2004
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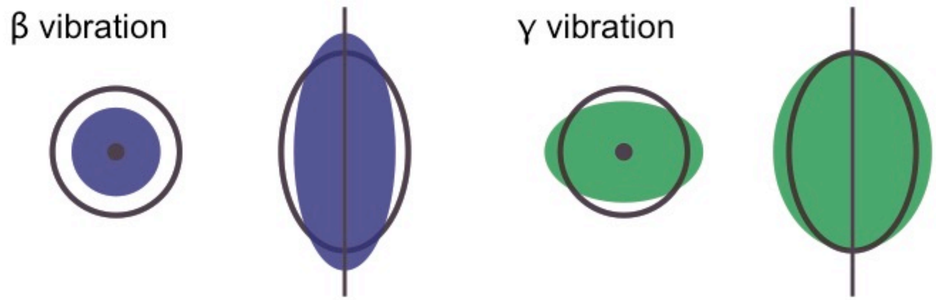
W.D. Kulp et al., Phys. Rev. C 77 061301R 2008
P.E. Garrett et al., Phys. Rev. Lett. 103 062501 2009
P.E. Garrett, J. Phys. G: Nucl. Part. Phys. 27 R1 2001 [Review]
J.L. Wood et al., Nucl. Phys. A 651 323 1999
Kris Heyde and John L. Wood, Rev. Mod. Phys. 83 1467 2011

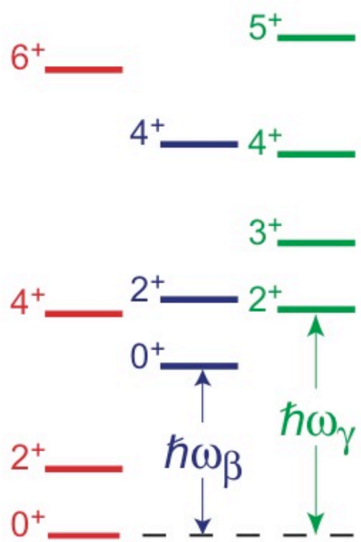
Simple ideas of low-energy quadrupole vibrations in deformed nuclei

Transitional nuclei have been described as "soft" deformed structures that have both rotational and vibrational modes of excitation.

Rotational bands are built on vibrational excitations of the ground state.

Two types of vibration:
axially-symmetric β vibrations;
asymmetric γ vibrations.





Low-energy quadrupole vibrations in deformed nuclei: simple multi-phonon patterns

Transitional nuclei have been described as "soft" deformed structures that have both rotational and vibrational modes of excitation.

Rotational bands are built on vibrational excitations of the ground state.

Two types of vibration:

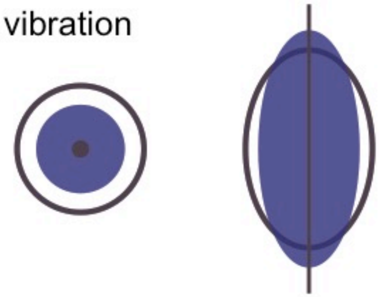
axially-symmetric β vibrations;

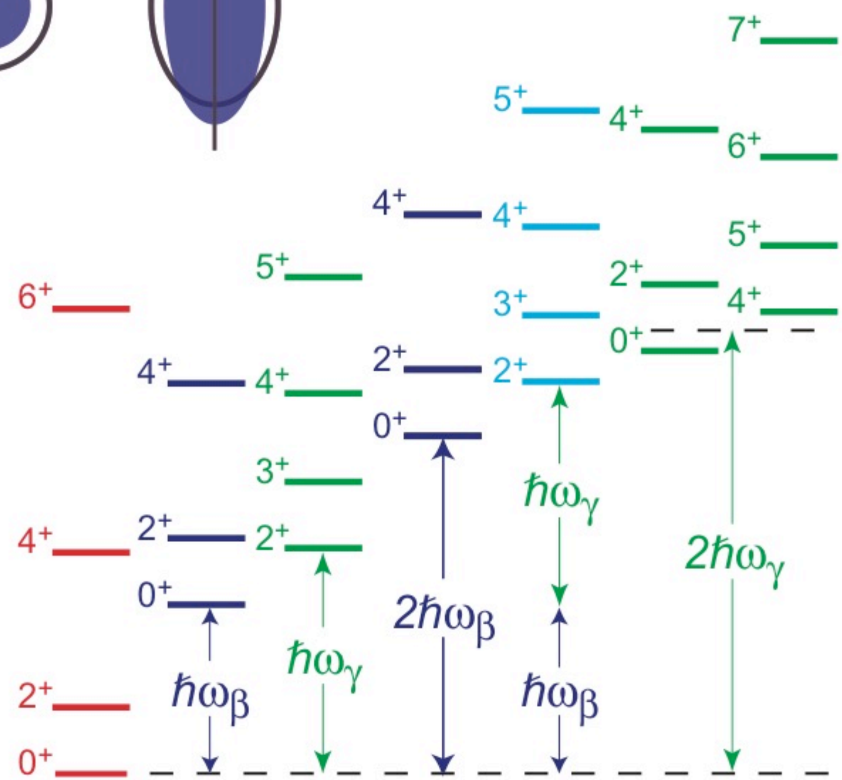
asymmetric γ vibrations.

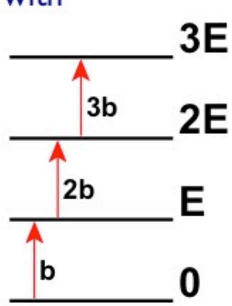
Multiple- and mixed-phonon states are expected at higher energies.

Transition strength scales with phonon number.

β vibration







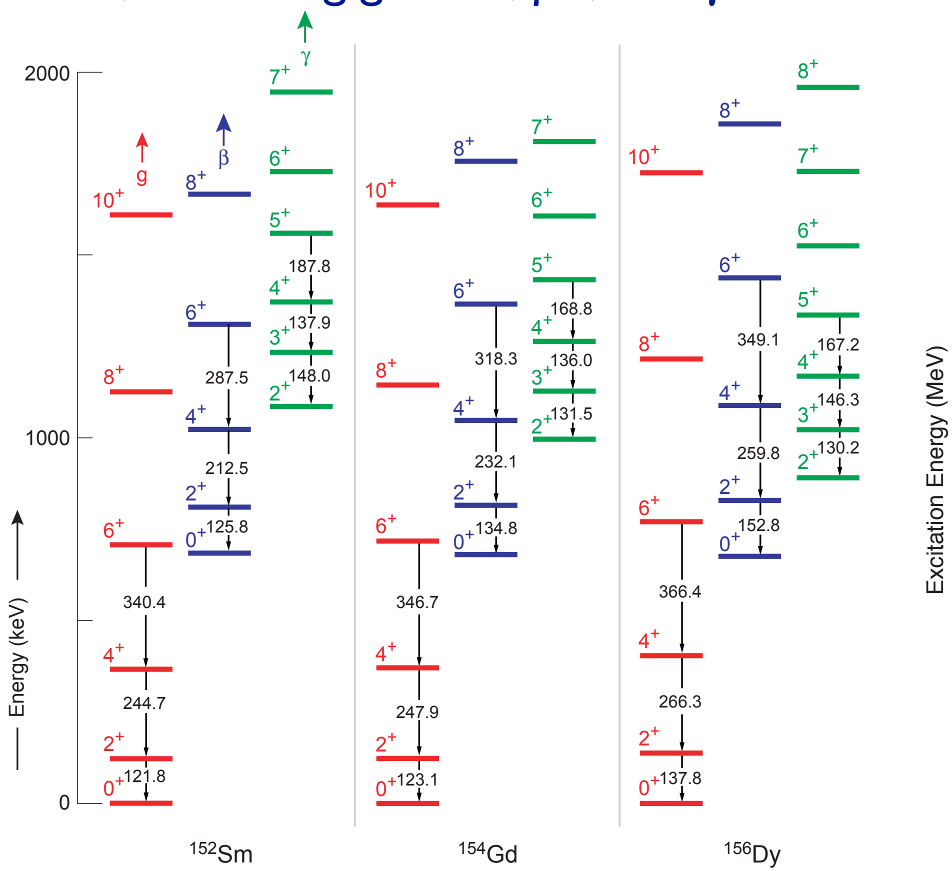
$$a^{\dagger}|0\rangle = |1\rangle$$

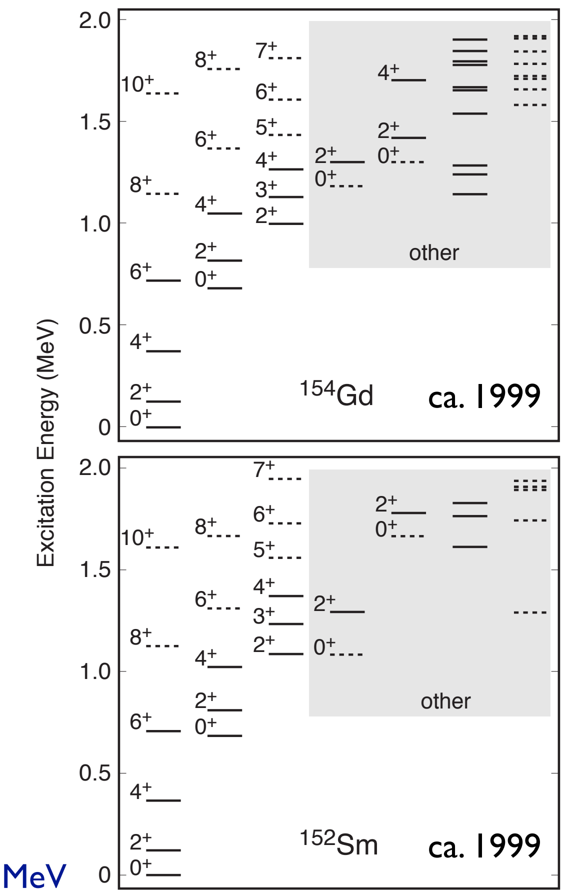
$$a^{\dagger}|n\rangle = \sqrt{n+1}|n+1\rangle$$

$$T(n \rightarrow n+1) = b\langle n+1|a^{\dagger}|n\rangle^2 = b(n+1)$$

Where are the best examples of low-energy quadrupole vibrations in deformed nuclei?

The N = 90 nuclei appear to be the quintessential collective model nuclei, exhibiting ground, β−, and γ−vibrational rotational bands.

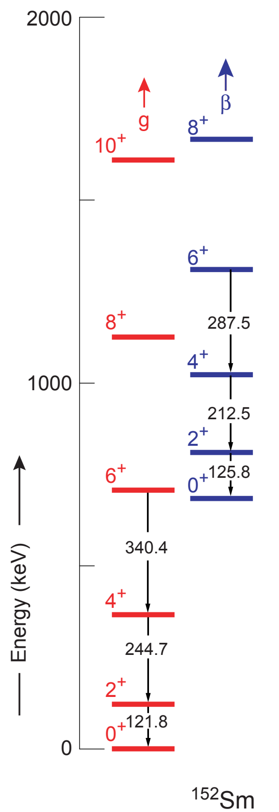




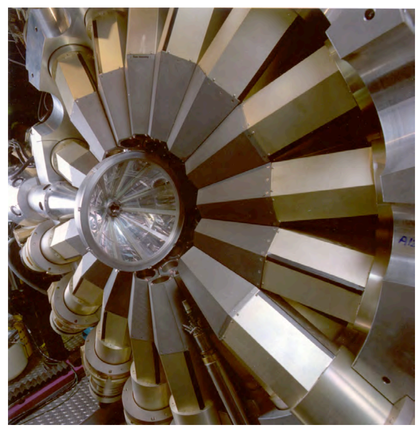
Among other states reported above 1 MeV were candidate 2-phonon excitations.

Where are the best examples of low-energy quadrupole vibrations in deformed nuclei?

The N = 90 nuclei appear to be the quintessential collective model nuclei, exhibiting ground, β−, and γ−vibrational rotational bands.



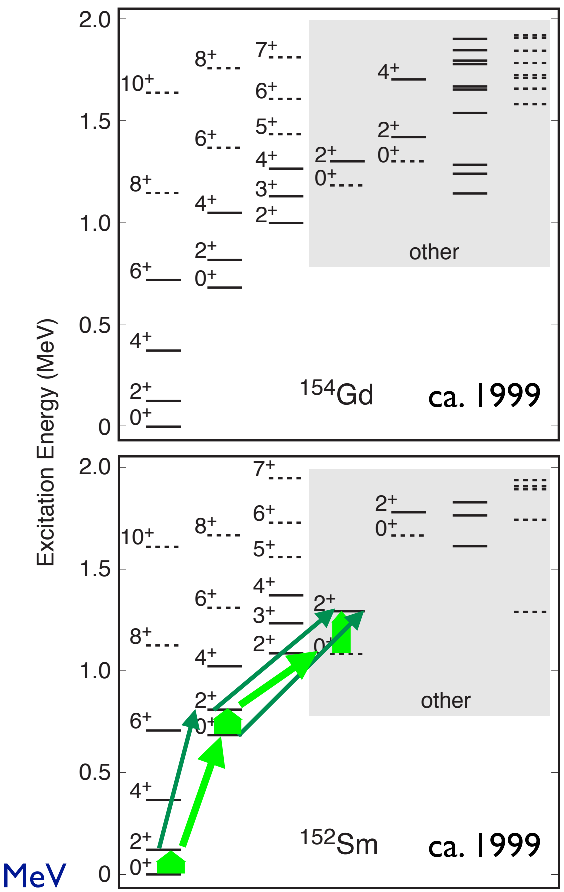
A two-phonon vibrational band should be readily identified through multiple-step Coulomb excitation.



Experiment carried out at the 88-inch Cyclotron Facility at LBNL using Gammasphere and CHICO.

62 hours

7×10^8 p-p-γ, 8×10^7 p-p-γ-γ, 10^7 p-p-γ-γ-γ

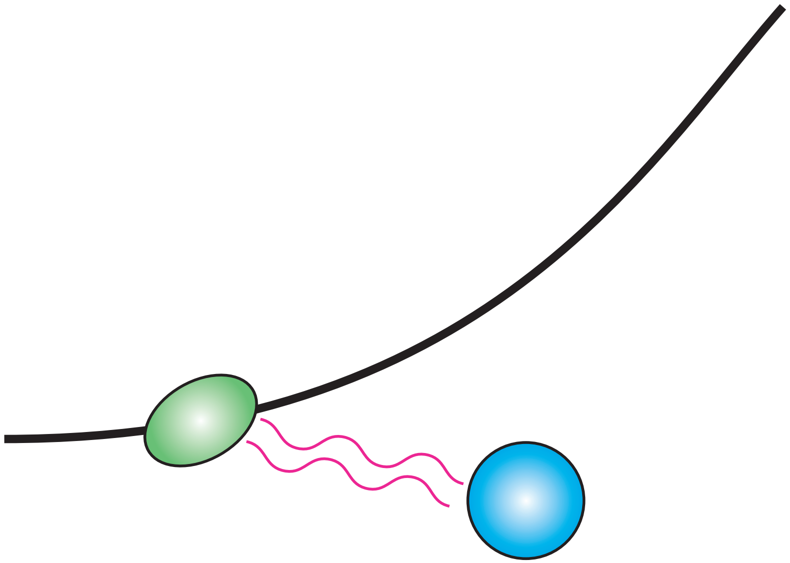


Among other states reported above 1 MeV were candidate 2-phonon excitations.

Multi-step Coulomb Excitation

Multiple-step Coulomb excitation‡ (multi-Coulex) populates excited collective states in nuclei. (‡ long-ranged ↔ collective excitation)

An incident nucleus is scattered by the Coulomb interaction with a target nucleus.



Multi-step Coulomb Excitation

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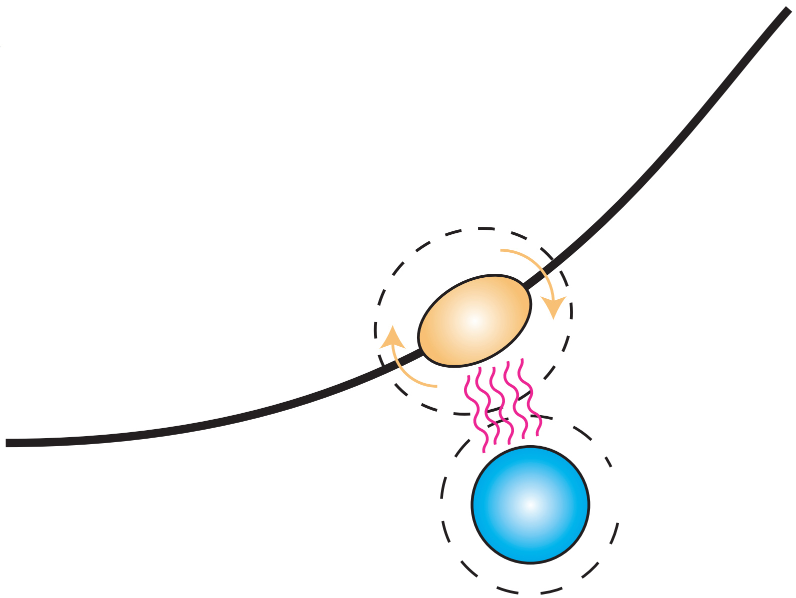
A close ("safe") approach results in multiple Coulomb interactions only (no complication from strong interactions).



Multi-step Coulomb Excitation

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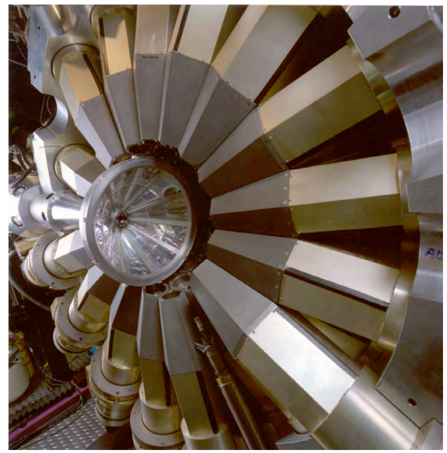
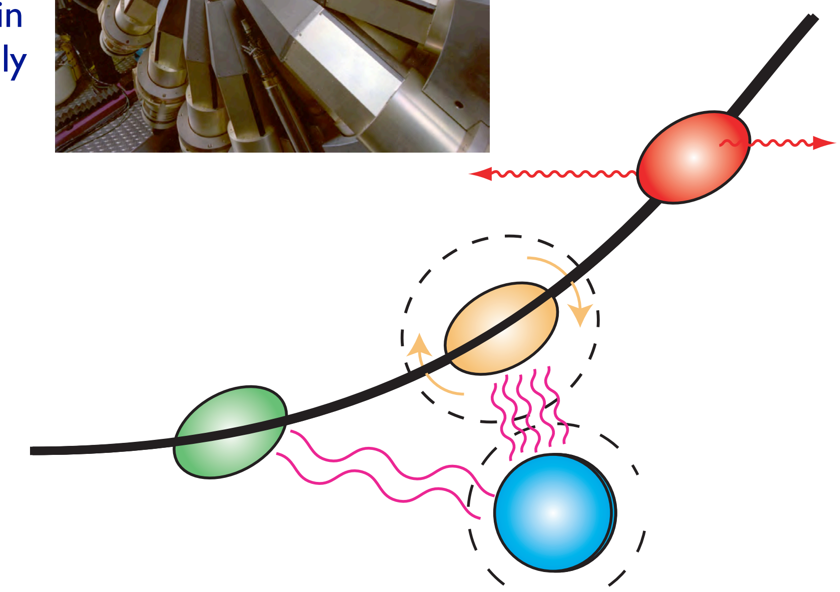
Level energies and transition strengths are deduced through γ-ray spectroscopy.

Level lifetimes, quadrupole moments and transition matrix elements are deduced by a least-squares fit to γ-ray yields.

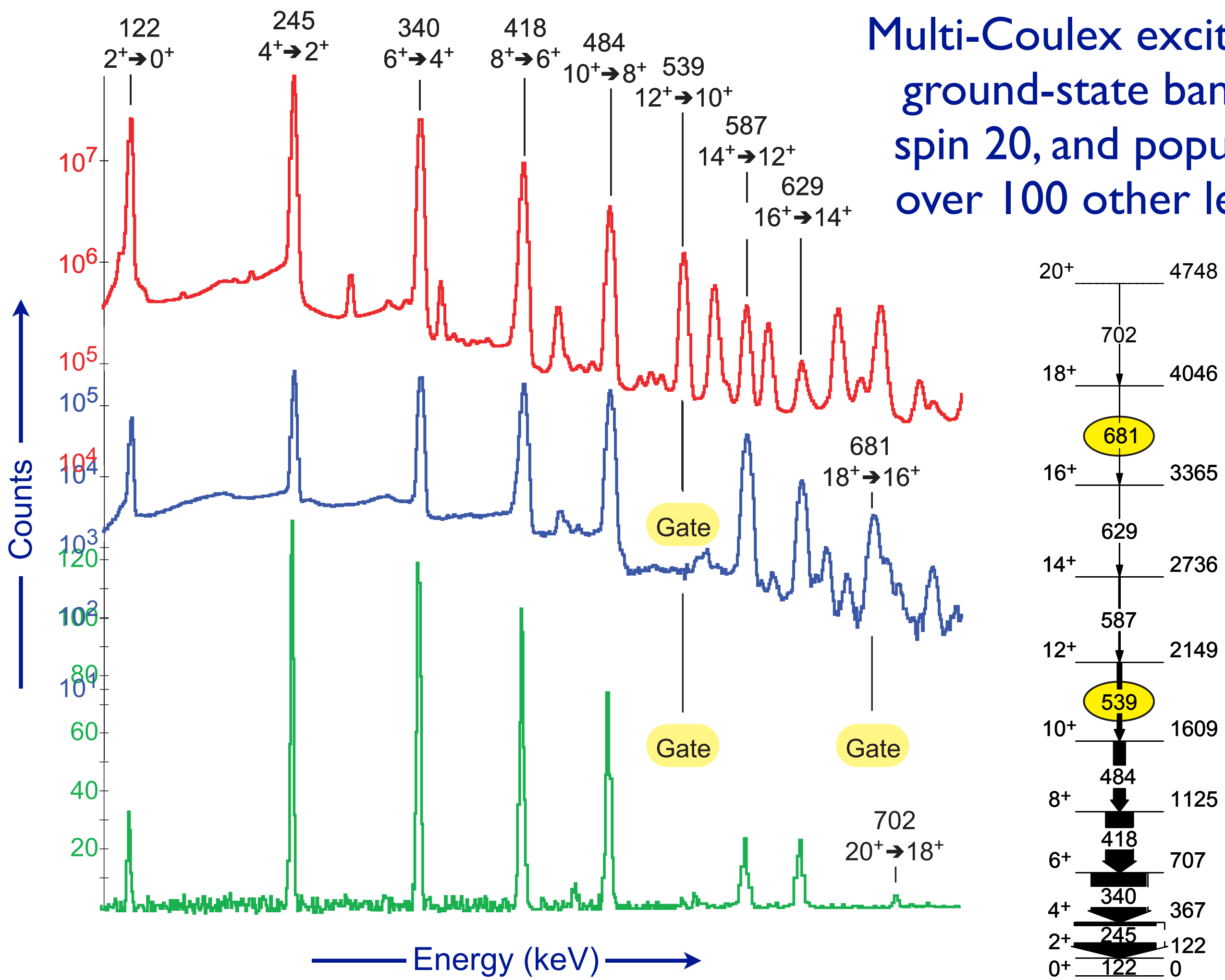


Gammasphere:
110 Ge γ-ray detectors.

CHICO:
particle detector.

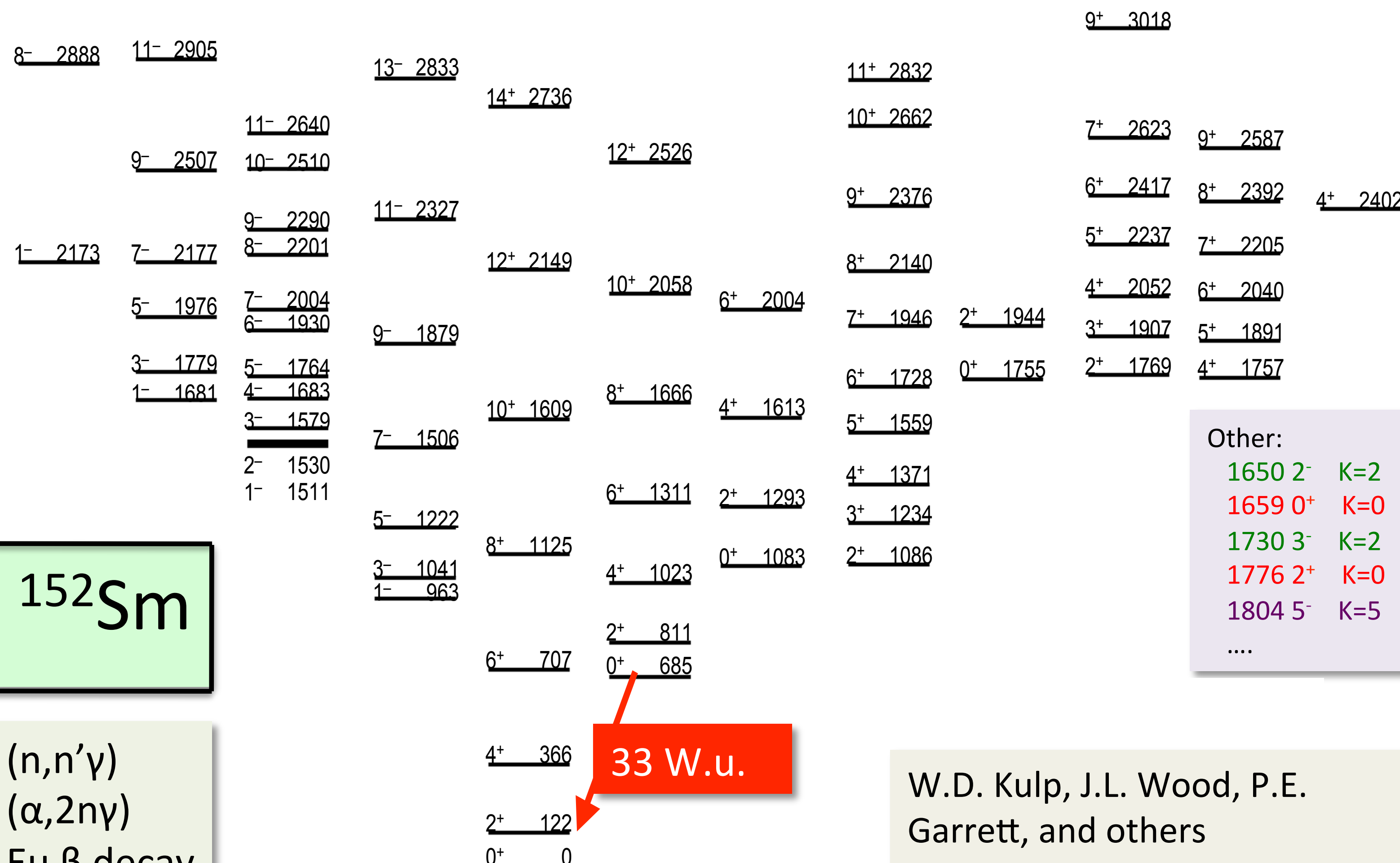


Multi-step Coulomb Excitation: ^{152}Sm



Multi-Coulex excites the ground-state band to spin 20, and populates over 100 other levels.

^{152}Sm: what is the nature of the 0_2^+ (685 keV) state?



Other:
1650 2^- K=2
1659 0^+ K=0
1730 3^- K=2
1776 2^+ K=0
1804 5^- K=5
....

^{152}Sm

(n,n'γ)
(α,2nγ)
Eu β decay
Coulex

W.D. Kulp, J.L. Wood, P.E. Garrett, and others

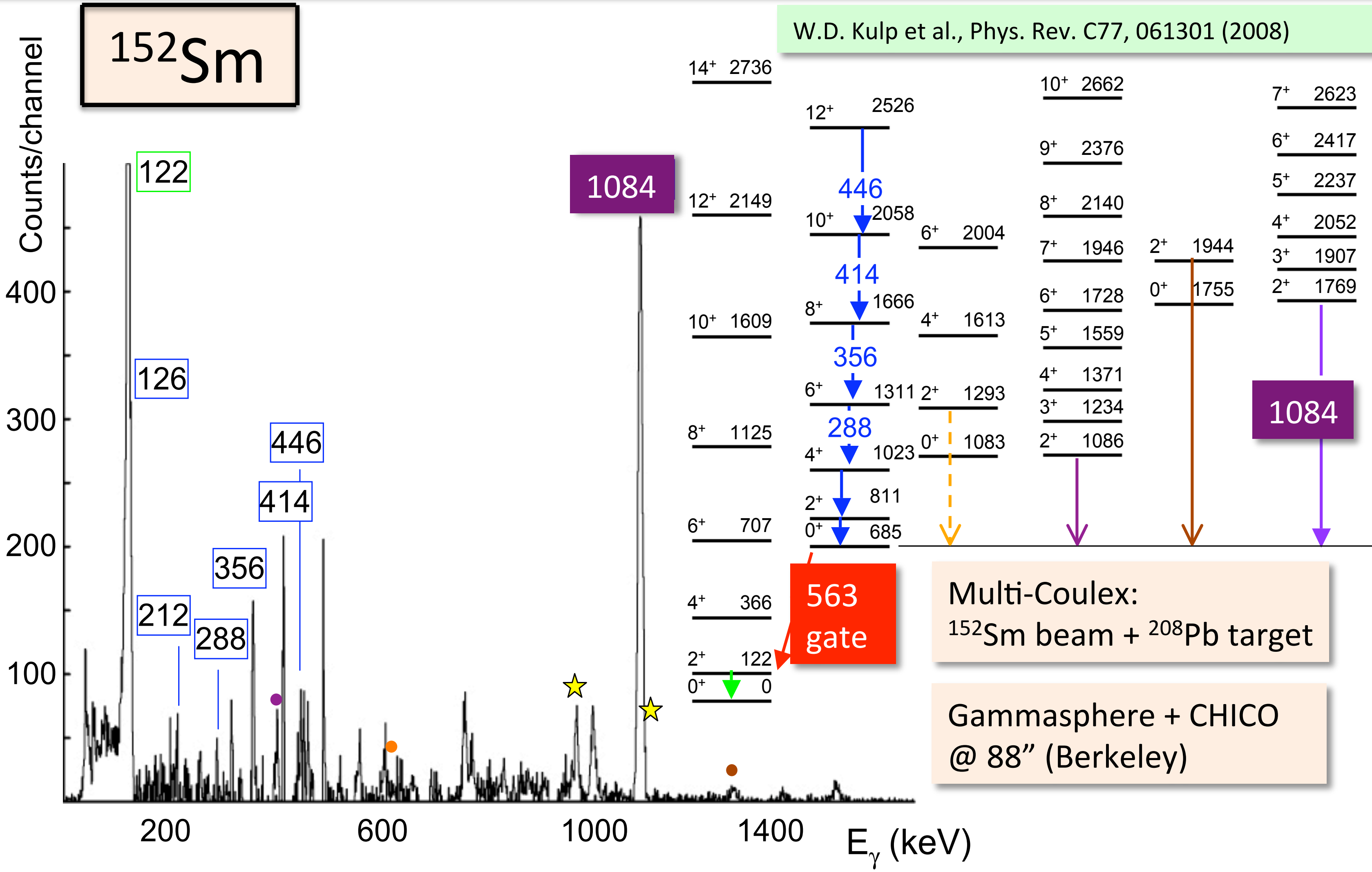
~400 states; Coulex ~200 states

Multi-Coulex of ^{152}Sm 0_2^+(685 keV):

strongest response is to head of K=2^+ band at 1769 keV

(in-band response attenuated by 99.7% decay out @ 811 level)

W.D. Kulp et al., Phys. Rev. C77, 061301 (2008)



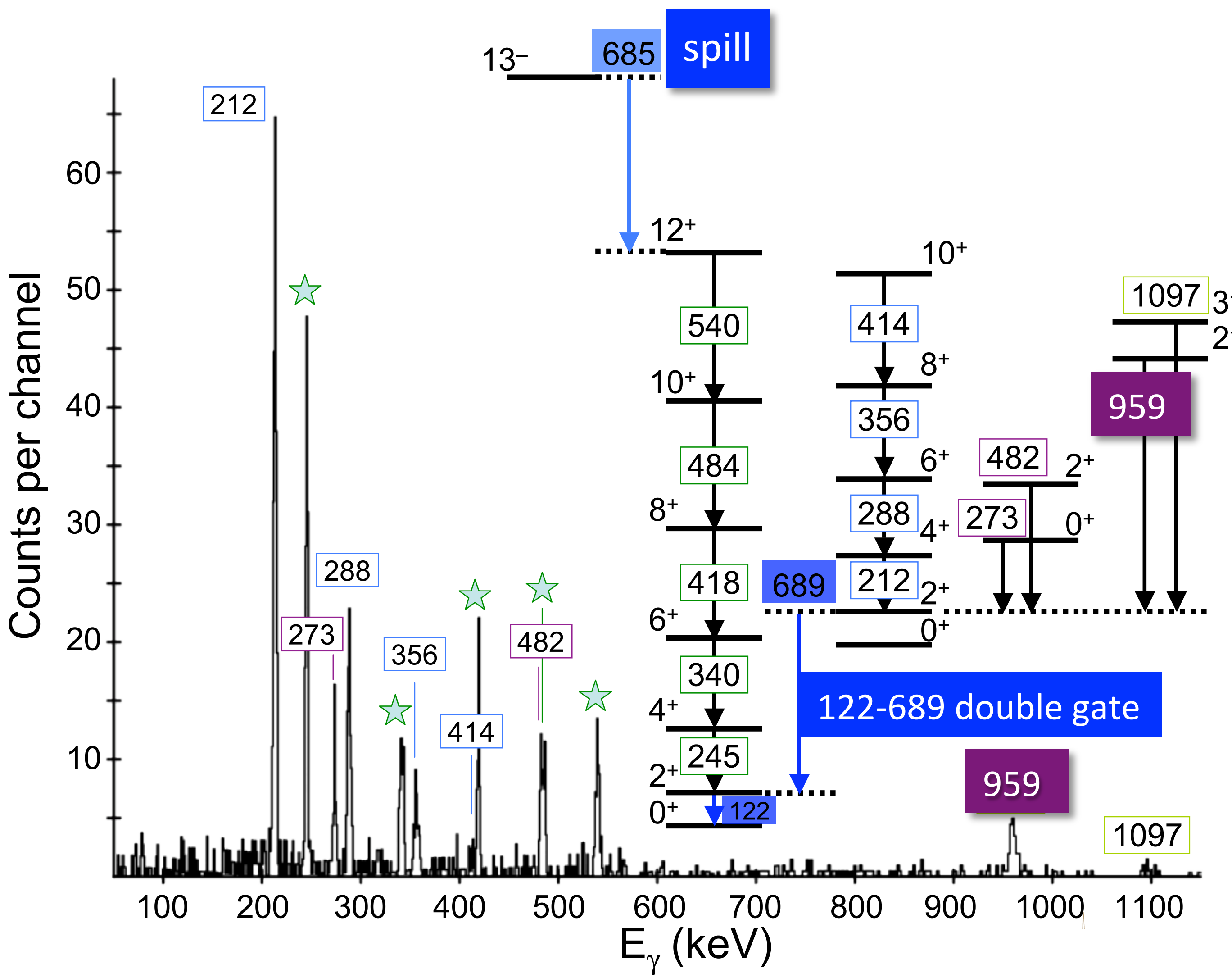
Multi-Coulex:
^{152}Sm beam + ^{208}Pb target

Gammasphere + CHICO
@ 88” (Berkeley)

Multi-Coulex of ^{152}Sm 2_2^+(811 keV):

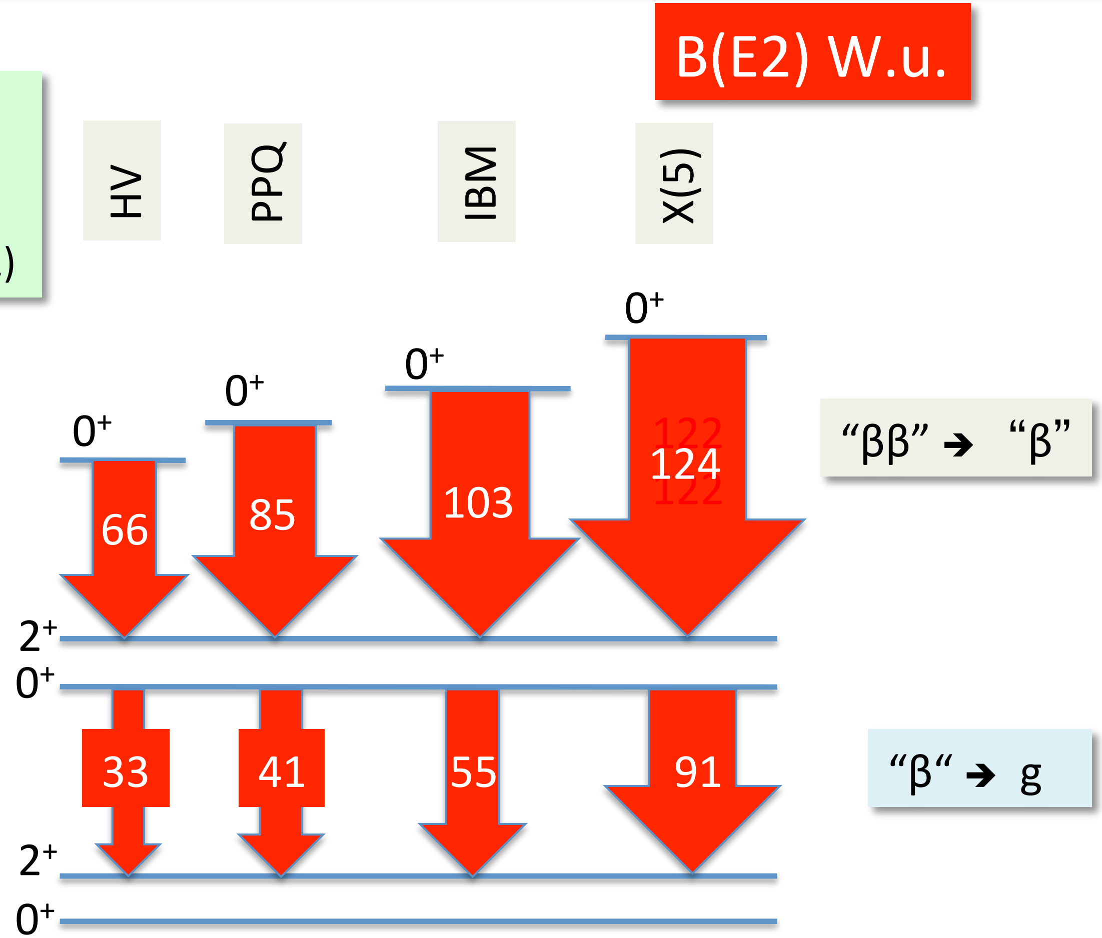
strongest responses are in-band and to K=2$^+$ band at 1769 keV

(in-band response attenuated by 92% decay out @ 4$^+$)

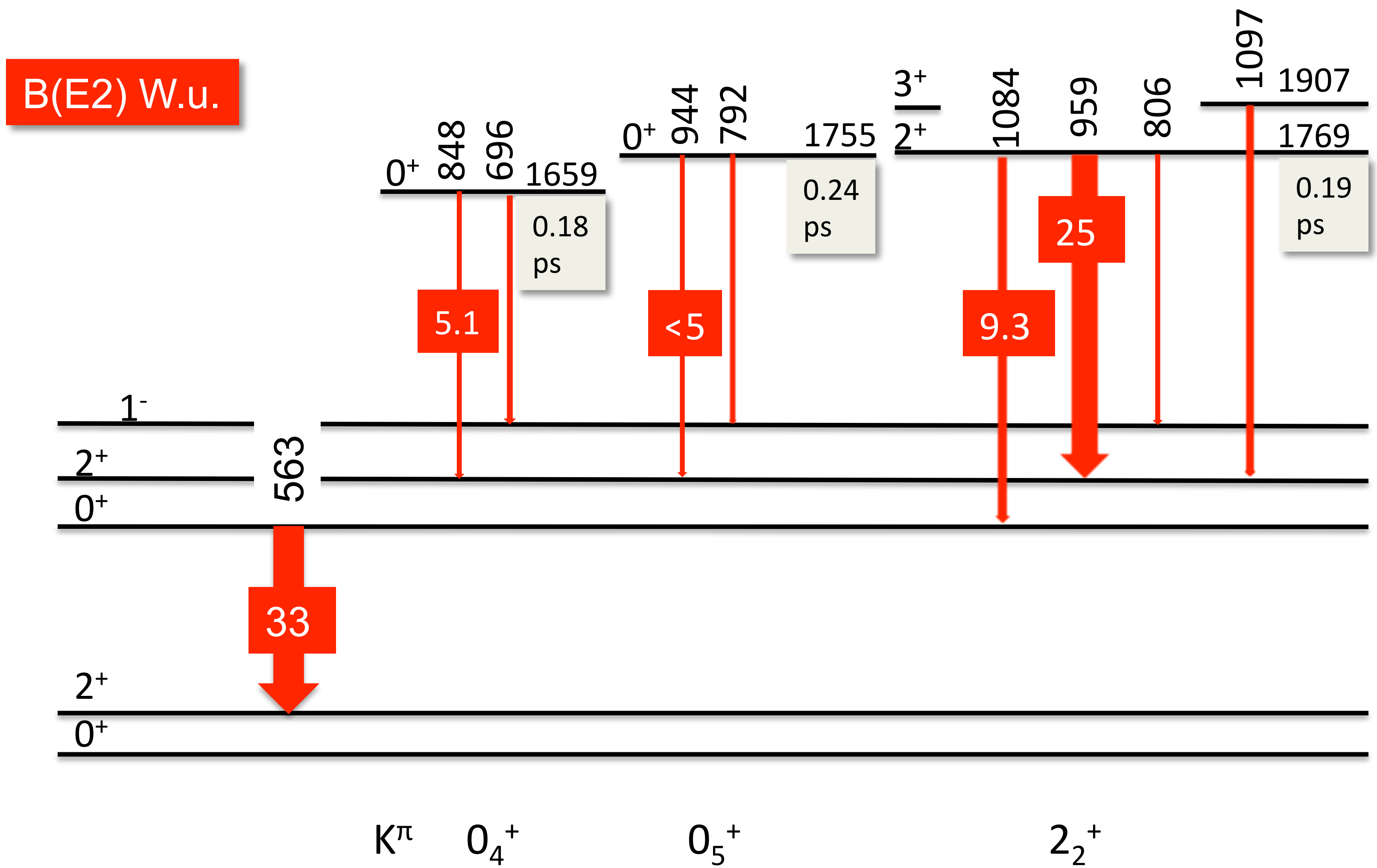


^{152}Sm: selected B(E2)'s theory

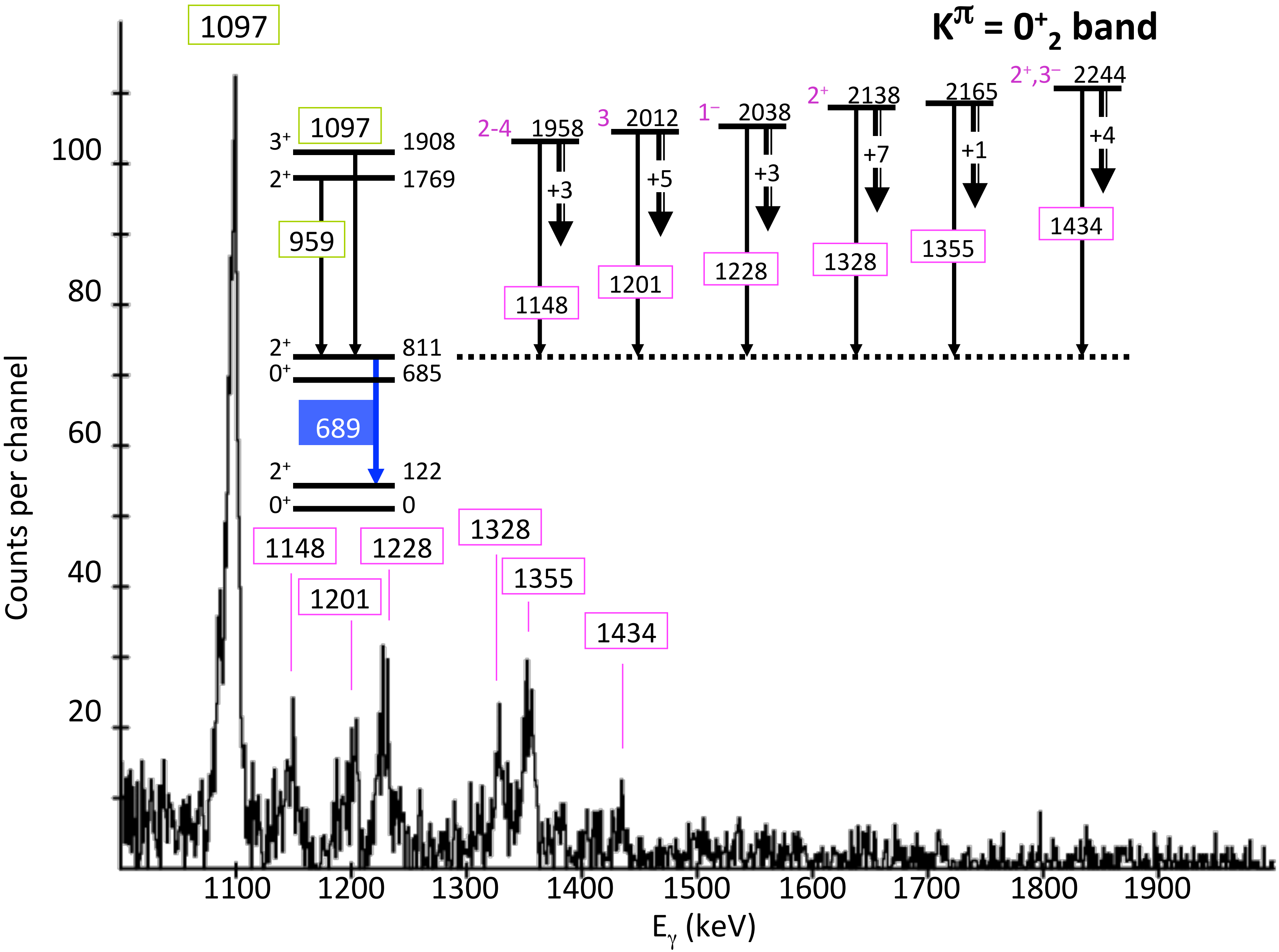
HV—harmonic vibrator
PPQ—Kumar, NP A231, 189 (1974)
IBM—Klug, PL B495, 55 (2000)
X(5)—Iachello, PRL 87, 052502 (2001)



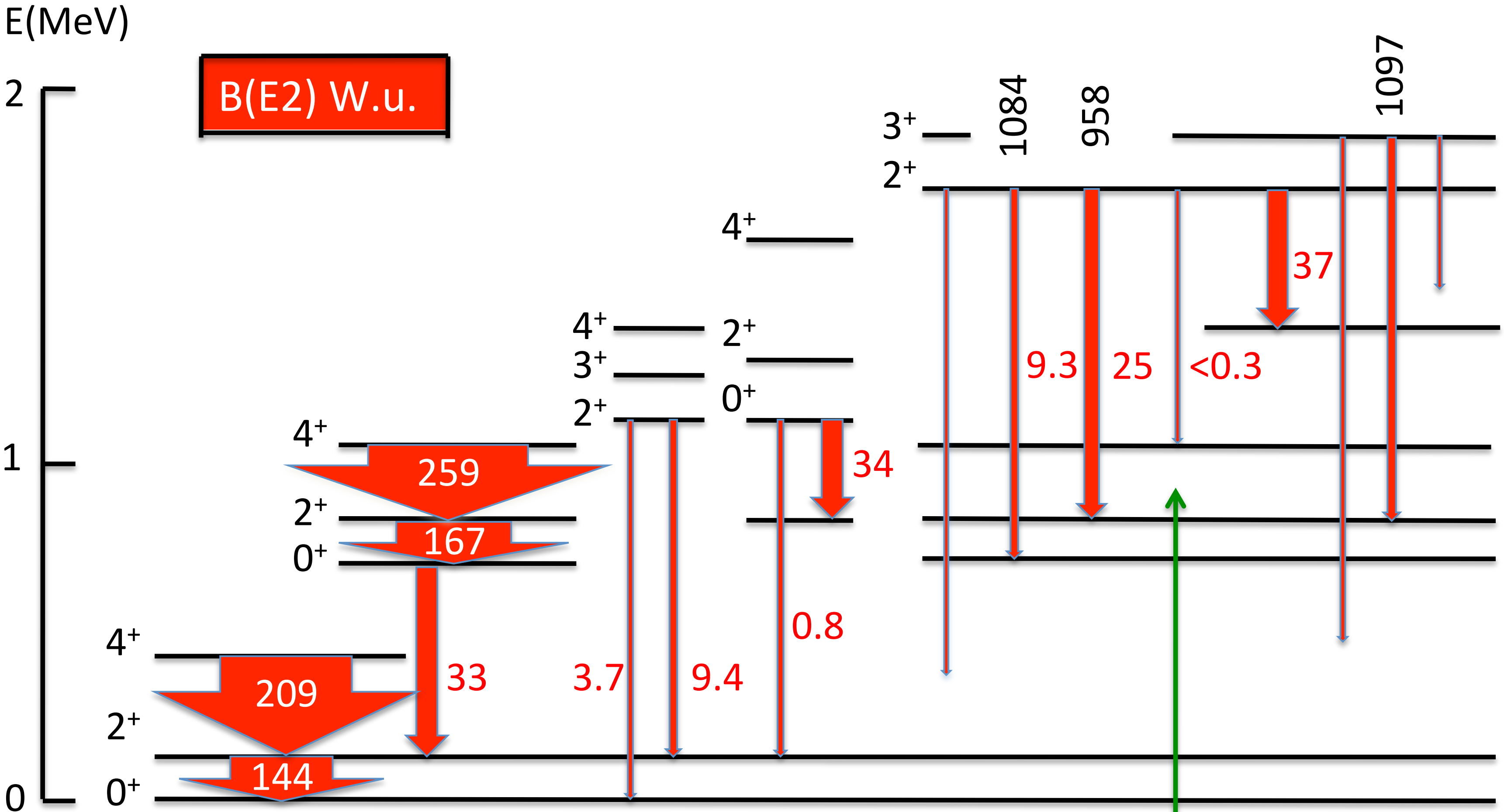
^{152}Sm: B(E2)' s to first excited $K^{\pi} = 0^{+}$ band



Collective strength to 2^+ state of $K^\pi = 0^+_2$ band



^{152}Sm: selected B(E2)'s expt.

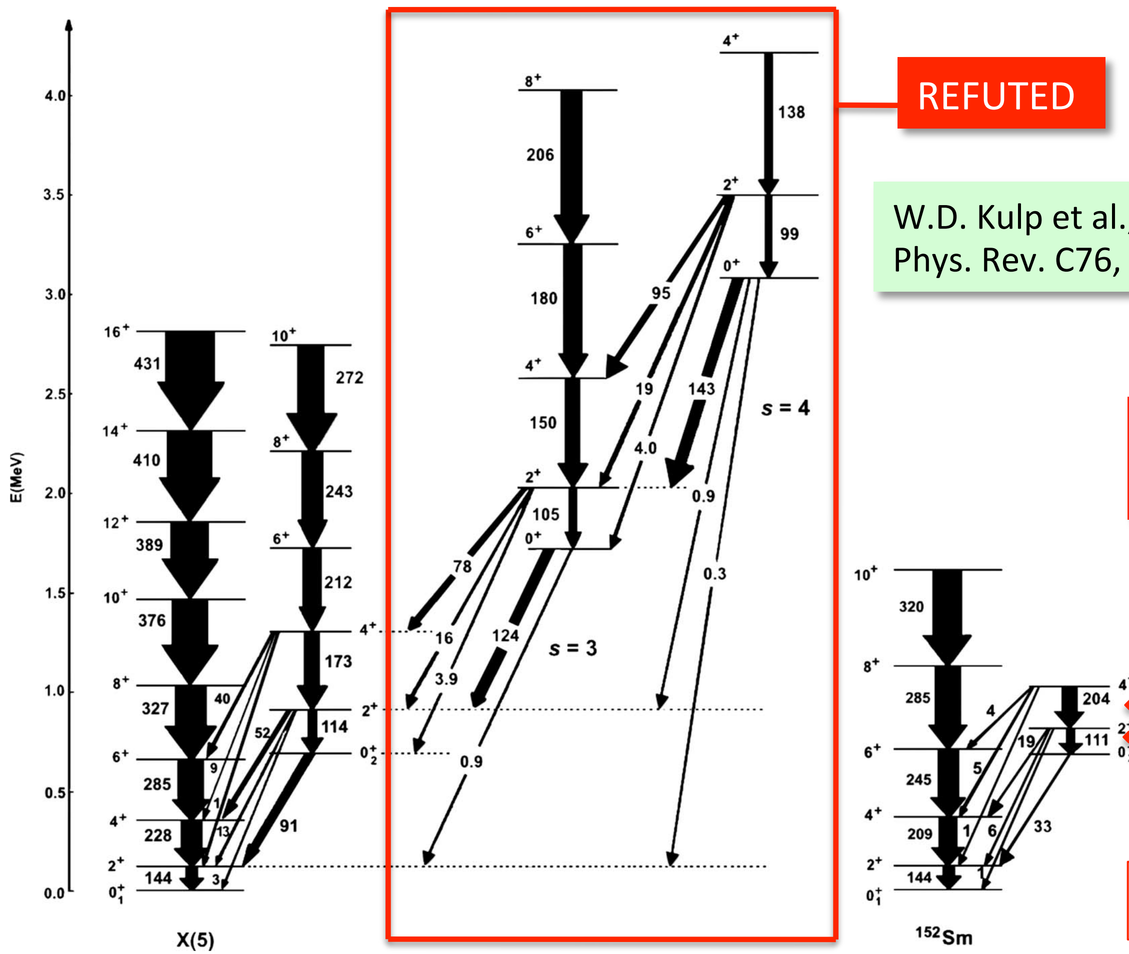


W.D. Kulp et al.: PR C71, 041303 (2005)
PR C76, 034319 (2007)
PR C77, 061301 (2008)

Alaga
K=2 1
K=0 45

^{152}Sm: prediction for collective quadrupole excitations in the X(5) model

Figure from P. Cejnar et al., Rev. Mod. Phys. 82, 2155 (2010)



REFUTED

W.D. Kulp et al.,
Phys. Rev. C76, 034319 (2007)

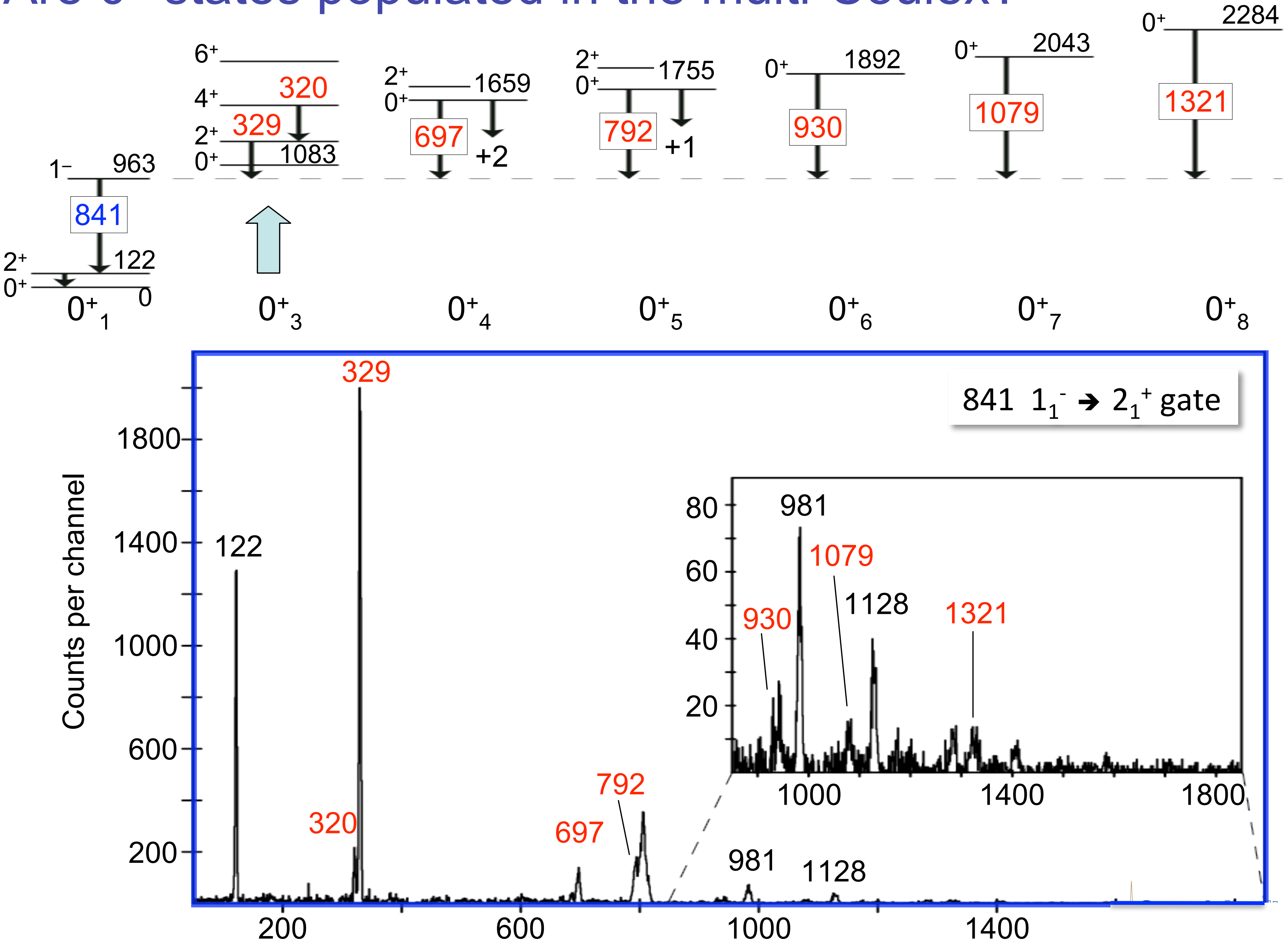
excited band is more deformed

260

167

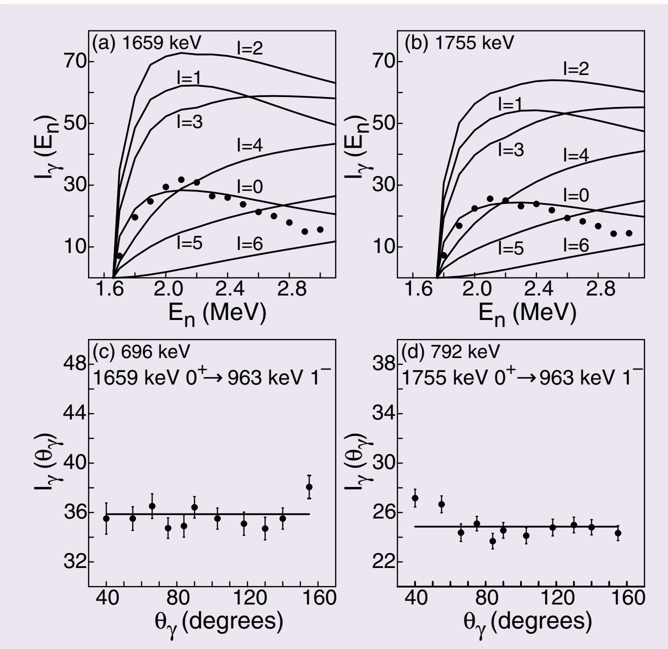
B(E2) W.u.

Are 0^+ states populated in the multi-Coulex?

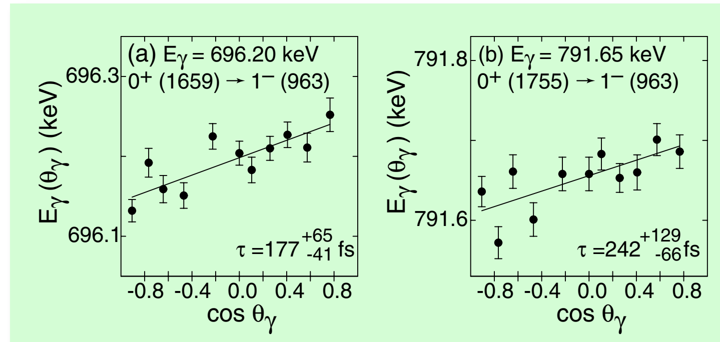


^{152}Sm: properties of 0_4^+(1659 keV) and 0_5^+(1755 keV) states from (n,n'γ)

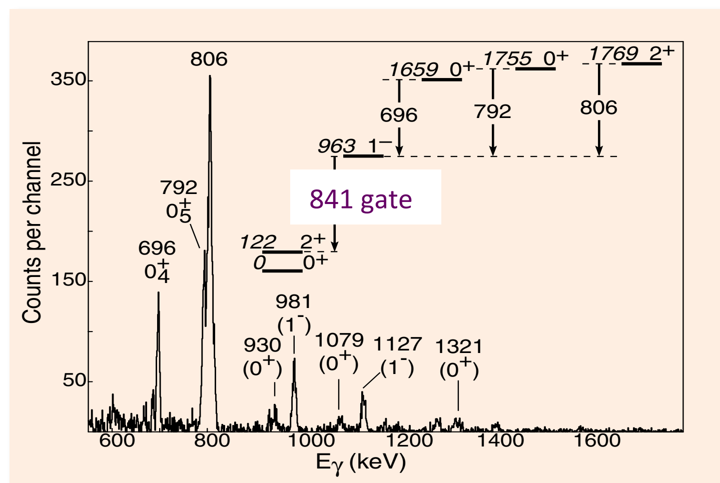
Spins from γ–ray excitation functions and angular distributions: (n,n'γ)



Lifetimes from Doppler induced γ–ray energy shifts: (n,n'γ)

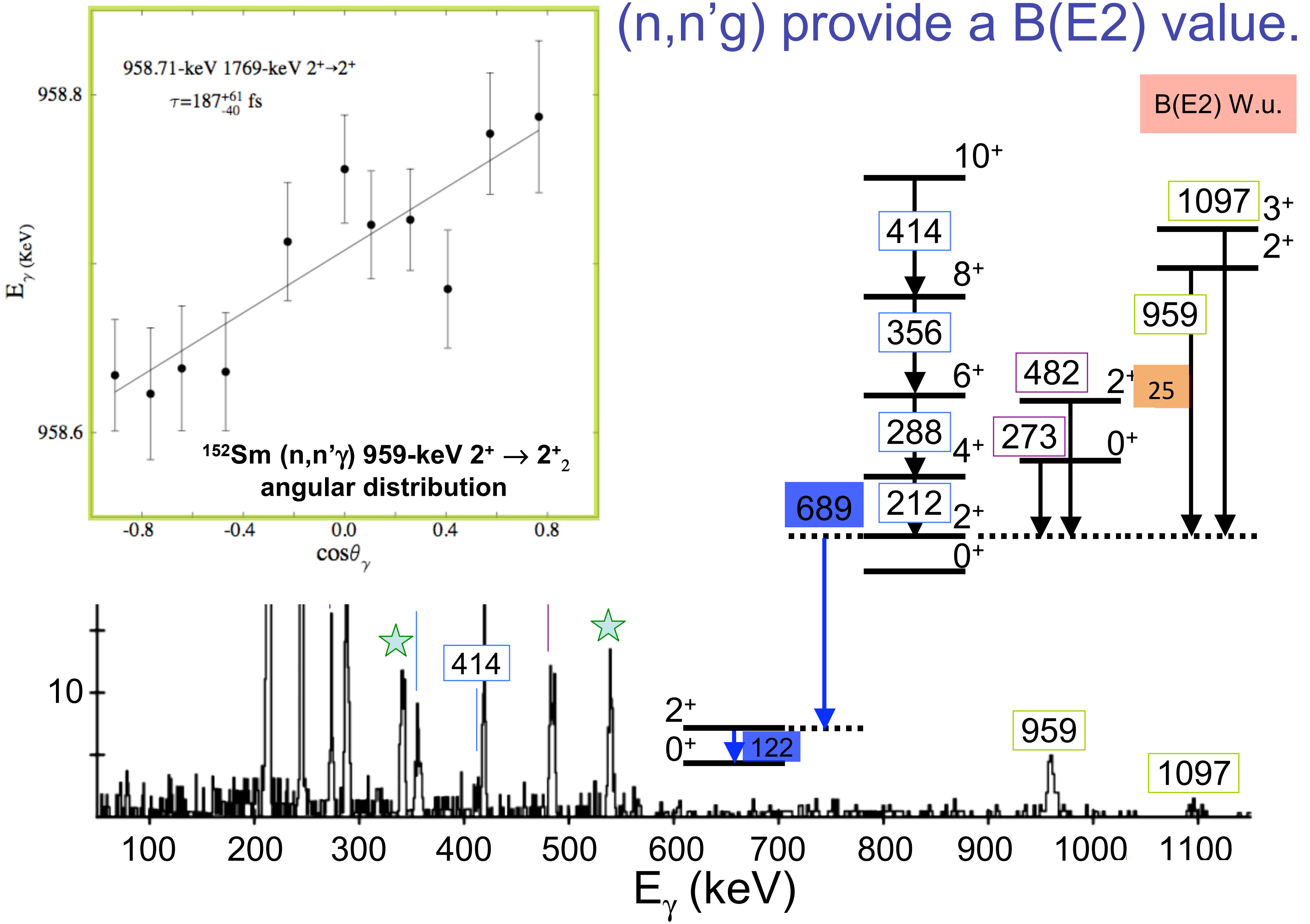


Coulex of higher energy 0^+ states seen via decay to the 1^- 963 keV state



W.D. Kulp et al.,
Phys. Rev. C77, 061301 (2008)

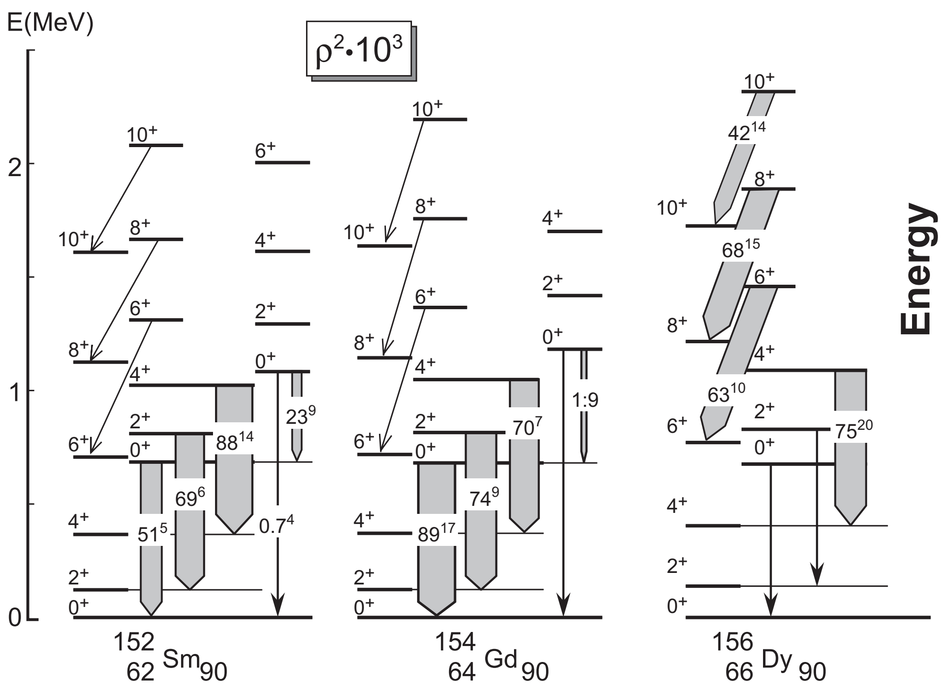
Lifetime measurements from (n,n'g) provide a B(E2) value.

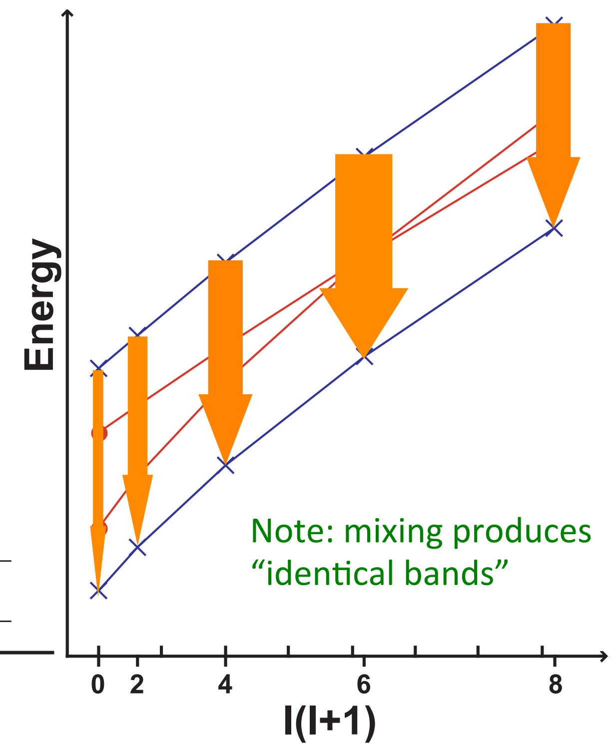


Shape coexistence in the N = 90 isotones:
revealed by E0 transition strengths

Strong mixing of coexisting shapes produces strong electric monopole (E0) transitions and identical bands.

Data from Heyde and Wood (2011)





$$\rho^2 \cdot 10^3 = \alpha^2\beta^2 \left(\Delta\langle r^2\rangle\right)^2 \cdot 10^3 \, \frac{Z^2}{R_0^4}$$

$$R_0 = 1.2A^{1/3}\ \mathrm{fm}$$

→ *E0* strength is a function of mixing.

Shape coexistence in the N = 90 isotones:

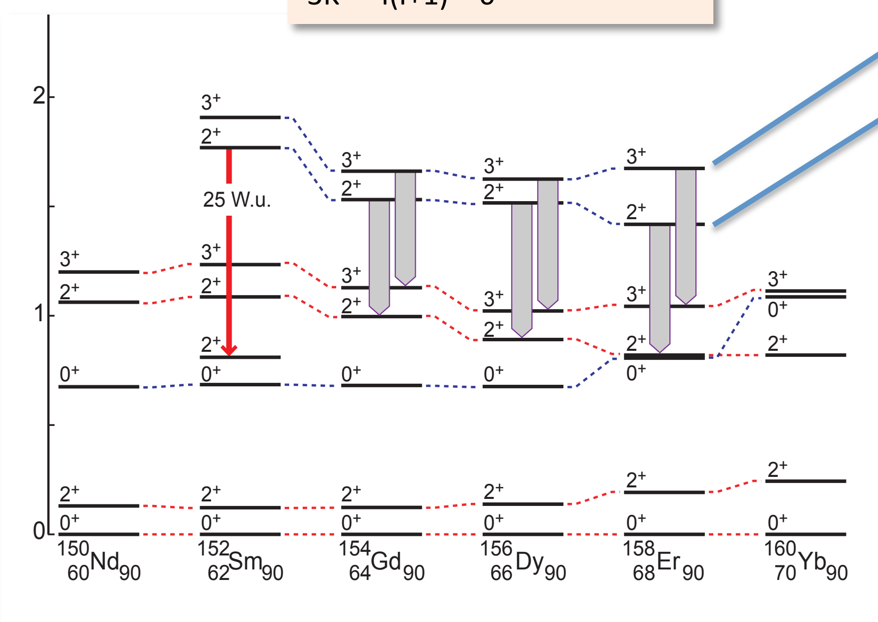
coexisting K = 2 bands revealed by E0 transitions

- Electric monopole transitions are a model-independent signature of shape coexistence and mixing (J.Kantele et al., Z. Phys. A289 157 (1979))

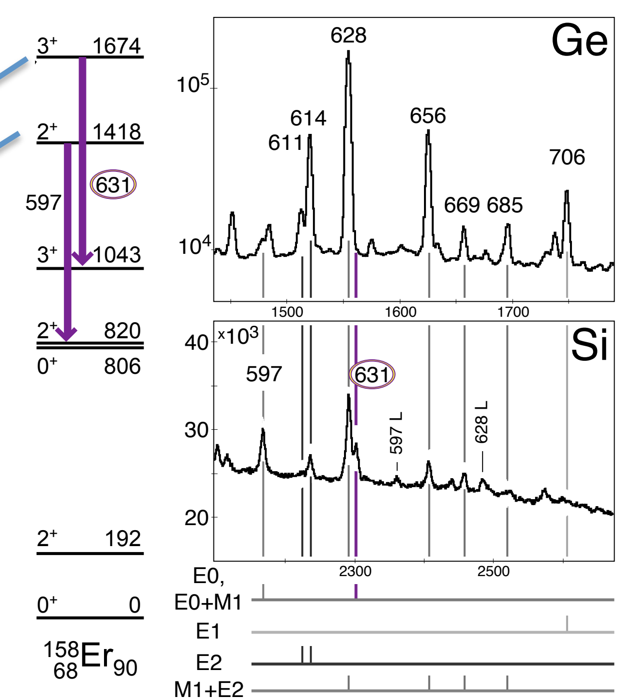
$3^+, K=2 \rightarrow 3^+, K=2$: no γ

$3K^2 - I(I+1) = 0$

E (MeV)



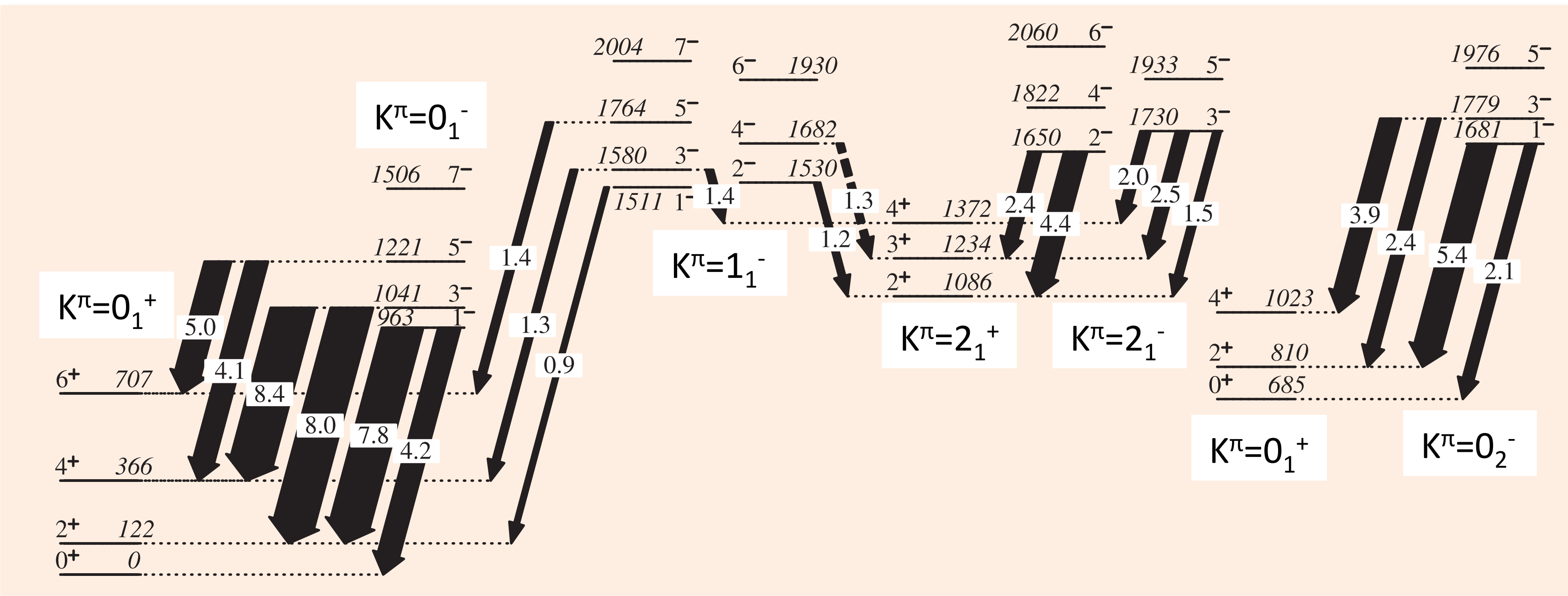
8π and PACES at TRIUMF/ISAC.



Kulp, Wood, Garrett, Zganjar and others

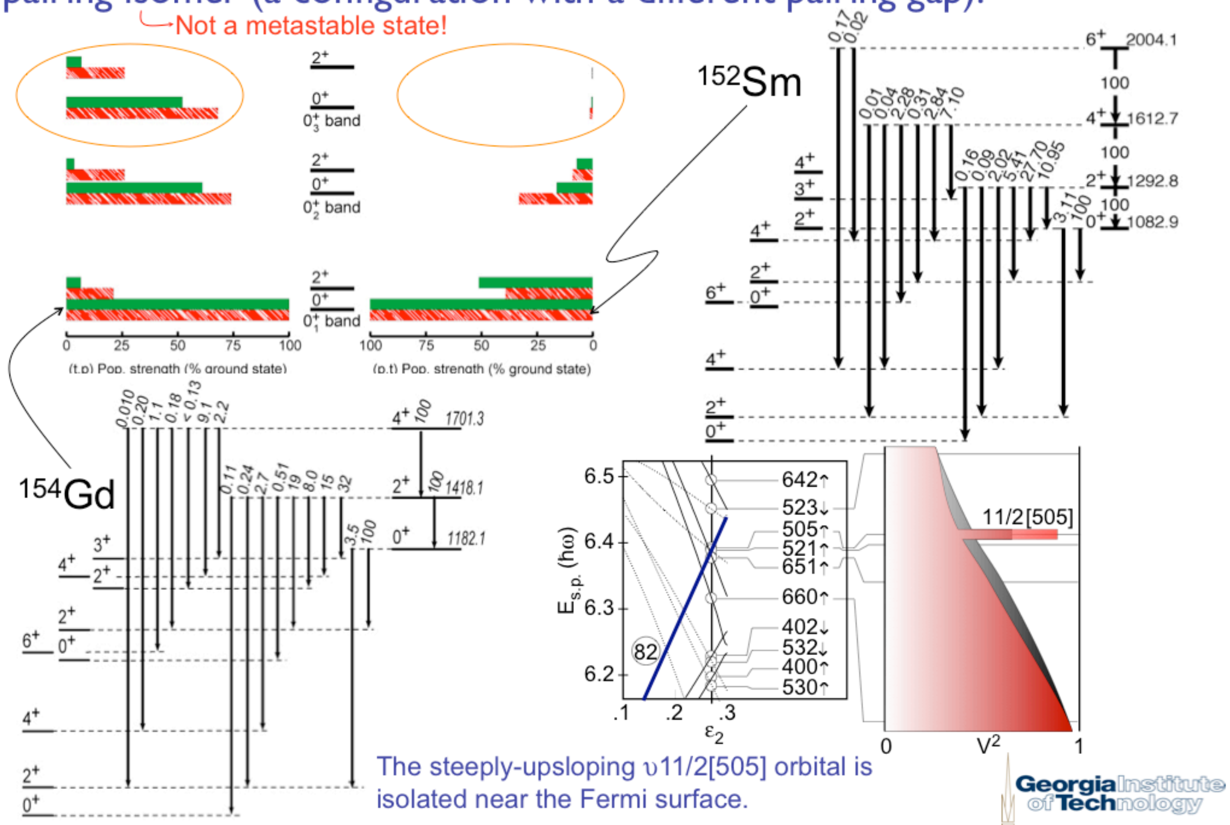
^{152}Sm: coexisting $K^{\pi} = 0^{-}$ bands

P.E. Garrett et al., Phys. Rev. Lett. 103, 062501 (2009)



Pairing isomers in the N = 90 isotones: ^{152}Sm and ^{154}Gd

Two-neutron transfer reactions reveal that the 0+3 state is a pairing isomer (a configuration with a different pairing gap).



The steeply-upsloping υ11/2[505] orbital is isolated near the Fermi surface.

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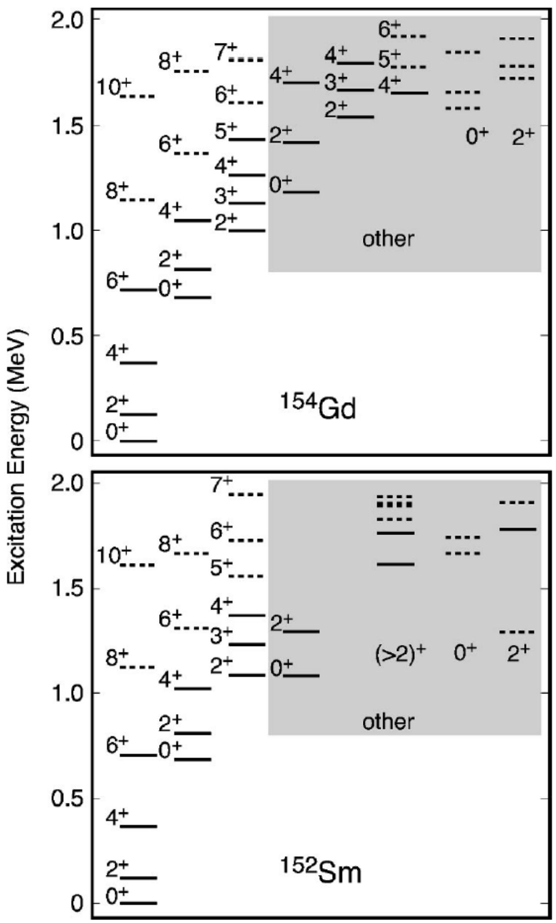
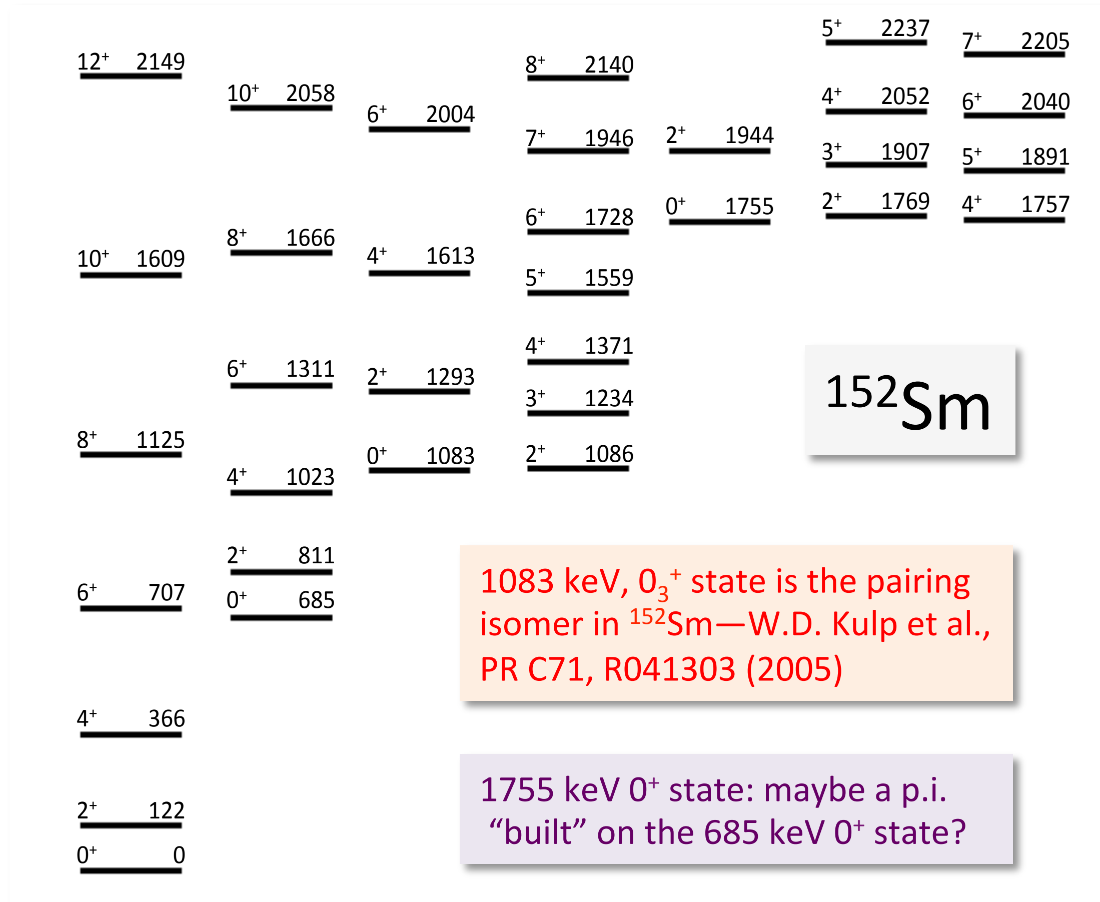
W.D. Kulp et al., Phys. Rev. Lett. 91, 102501 (2003)

0^+ pairing isomers @ N = 90:

0_3^+ states in ^{152}Sm and ^{154}Gd



^{152}Sm, ^{154}Gd, ca. 2003



1083 keV, 0_3^+ state is the pairing isomer in ^{152}Sm—W.D. Kulp et al., PR C71, R041303 (2005)

1755 keV 0^+ state: maybe a p.i. “built” on the 685 keV 0^+ state?

^{152}Sm, ca. 2008

Pairing isomerism mechanism

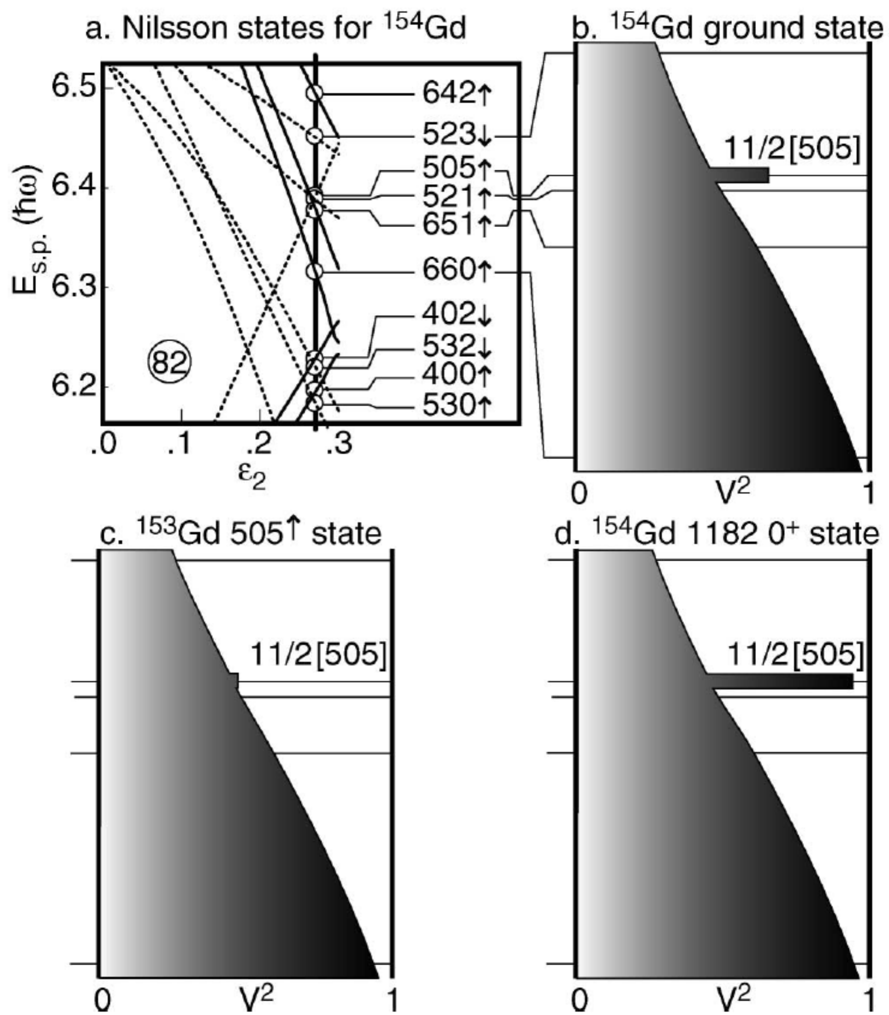
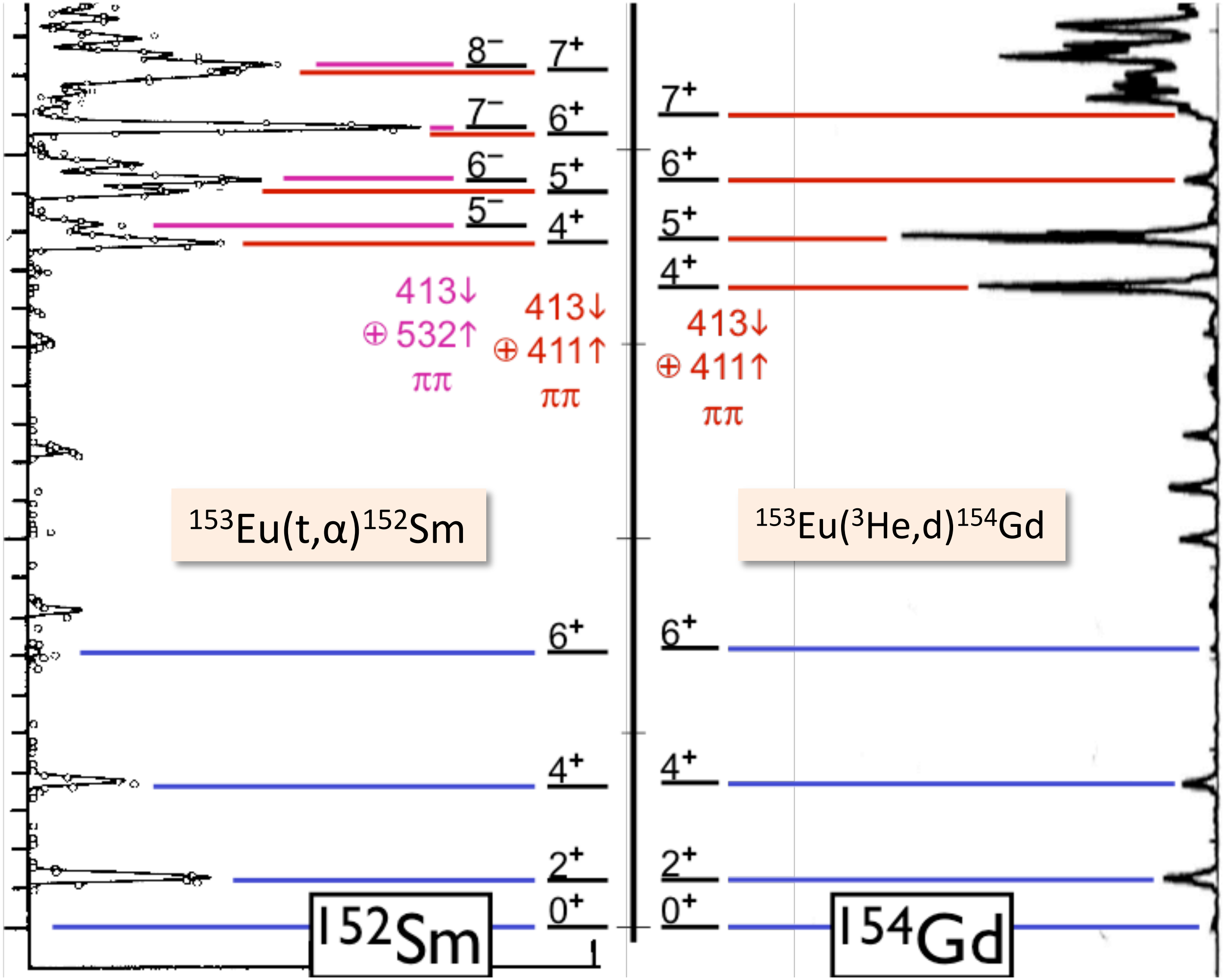


Figure: W.D. Kulp et al., PRL 91 102501 (2003)

Proximity of up-sloping and down-sloping Nilsson orbitals, as a function of deformation, between which there are reduced off-diagonal pairing matrix elements, results in two pairing condensates with very different pair occupancies, V^2, for at least one orbital ($11/2^-$ [505] @ N = 90).

"Pairing isomers", I. Ragnarsson and R.A. Broglia, NP A263 315 (1976)

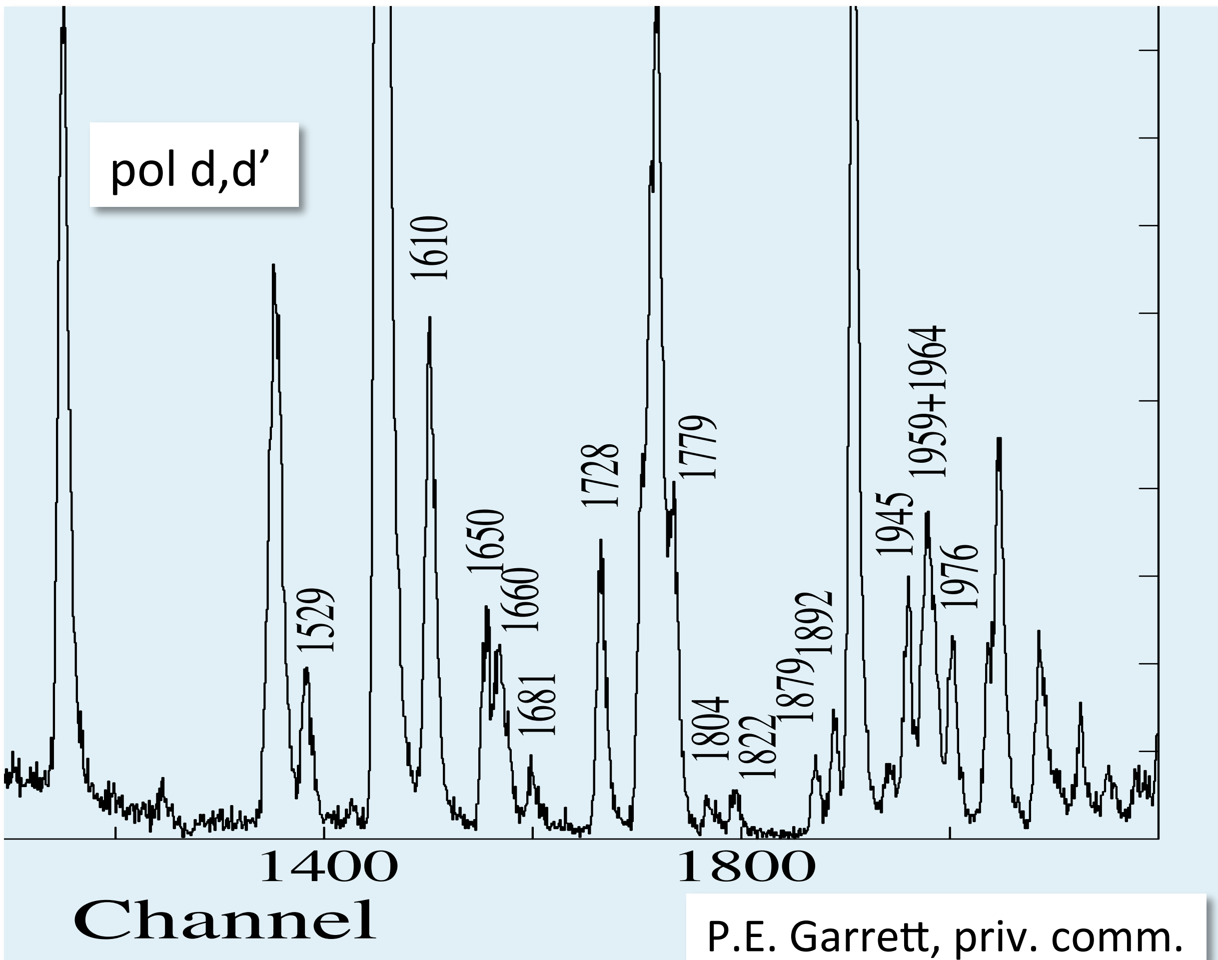
Nature of the $K^{\pi}=4^{+}_{1}$ band at N=90



D. Burke, et al., CJP 55, 1137 (1977)

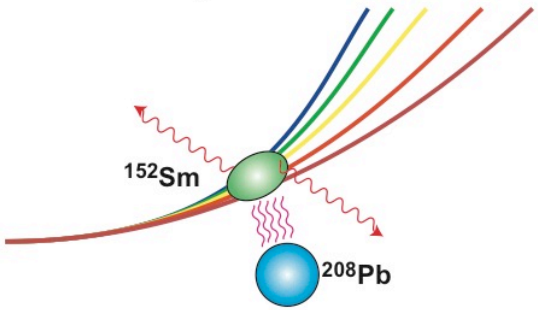
D. Burke, PRL 73, 1899 (1994)

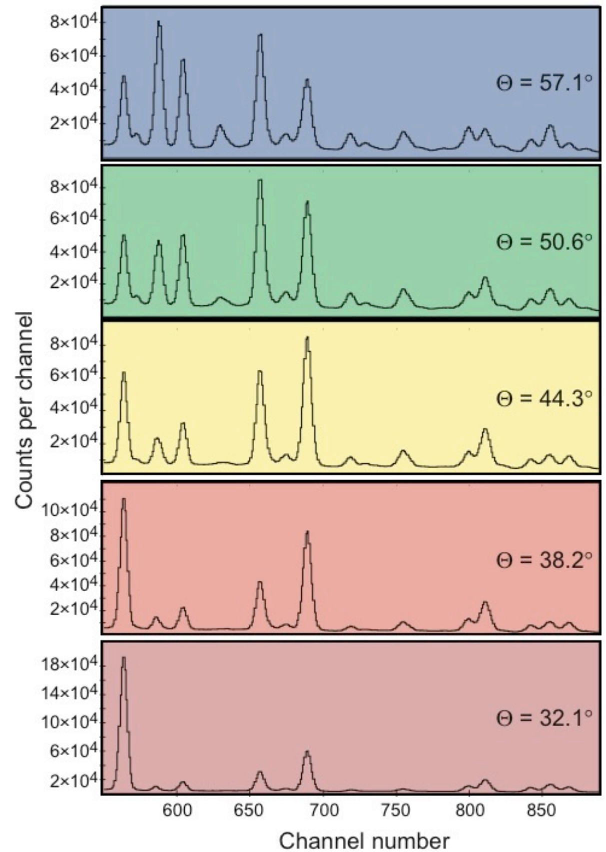
^{152}Sm: (pol d,d’) populates ALL states observed below ~ 1.88 MeV



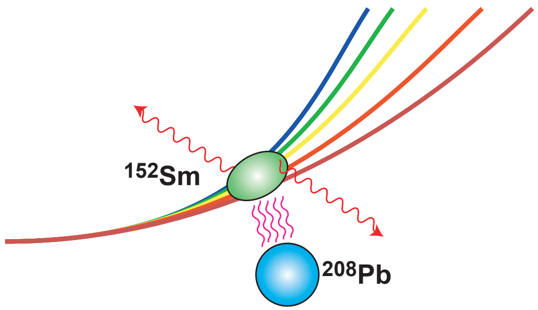
1506	7^-	✓	1822	4^-	✓
1511	1^-	✓	1879	9^-	✓
1529	2^-	✓	1891	5^+	
1559	5^-	✓	1892	0^+	✓
1570	3^-	✓	1906	2^+	✓
1609	10^+	✓	1906		
1613	4^+	✓	1908	3^+	
1650	2^-	✓	1920	6^-	
1660	0^+	✓	1930	6^-	
1666	8^+	✓	1933	5^-	✓
1681	1^-	✓	1944	1^-	✓
1682	4^-	✓	1945	2^+	✓
1728	6^+	✓	1946	7^+	
1730	3^-	✓	1946	0^+	✓
1755	0^+	✓	1954	5^-	✓
1757	4^+	✓	1958		✓
1764	5^-	✓	1964	1^-	✓
1769	2^+	✓	1964	4^+	✓
1776	2^+	✓	1977	5^-	✓
1779	3^-	✓			
1804	5^-	✓			

Angular selectivity results in multiple simultaneous experiments





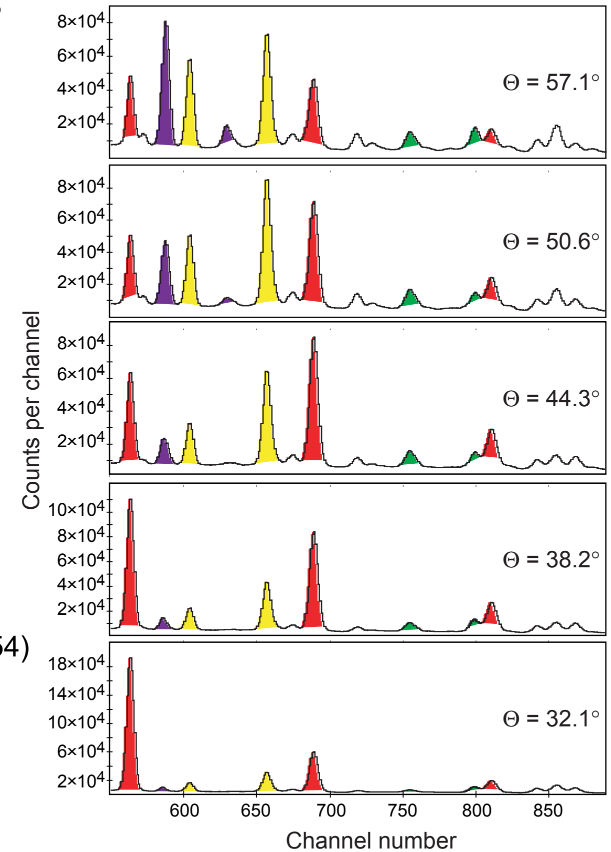
Angular selectivity results in multiple simultaneous experiments



Population of sideband low-spin states
0^+_β (563), 2^+_β (688, 810)

Population of medium-spin states
4^+_β (656), 6^+_β (604), 7^-_1 (799), 9^-_1 (754)

Population of high-spin states
14^+_g (587), 16^+_g (629)



Using Kumar-Kline sum rules, moment centroids and widths can be extracted from multi-Coulex matrix elements.

Quadrupole moment

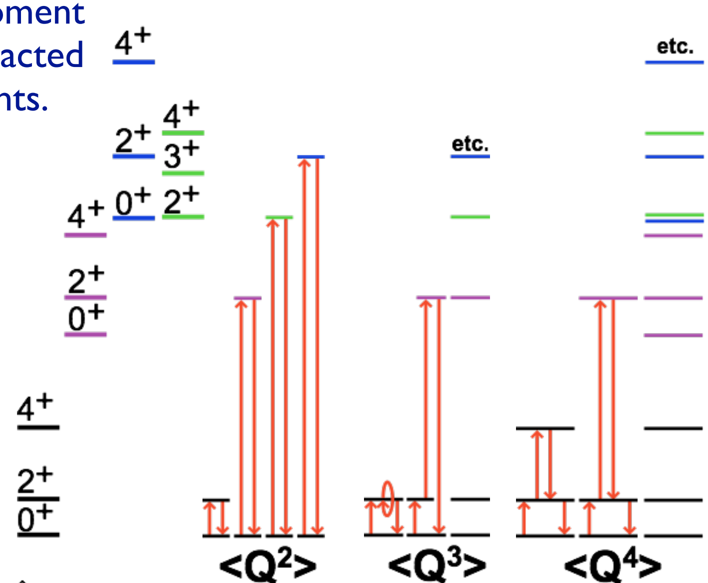
$$Q_0 = \langle q^2 \rangle^{\frac{1}{2}}$$

$$\sigma(q^2) = \sqrt{\langle q^4 \rangle - \langle\, q^2 \rangle}$$

Triaxiality

$$\gamma = \frac{1}{3} \arccos \frac{\langle q^3 \cos 3\delta \rangle}{\langle q^2 \rangle^{\frac{3}{2}}}$$

$$\sigma(q^3 \cos 3\delta) = \sqrt{\langle q^6 \cos^2 3\delta \rangle - \langle\, q^3 \cos 3\delta \rangle}$$

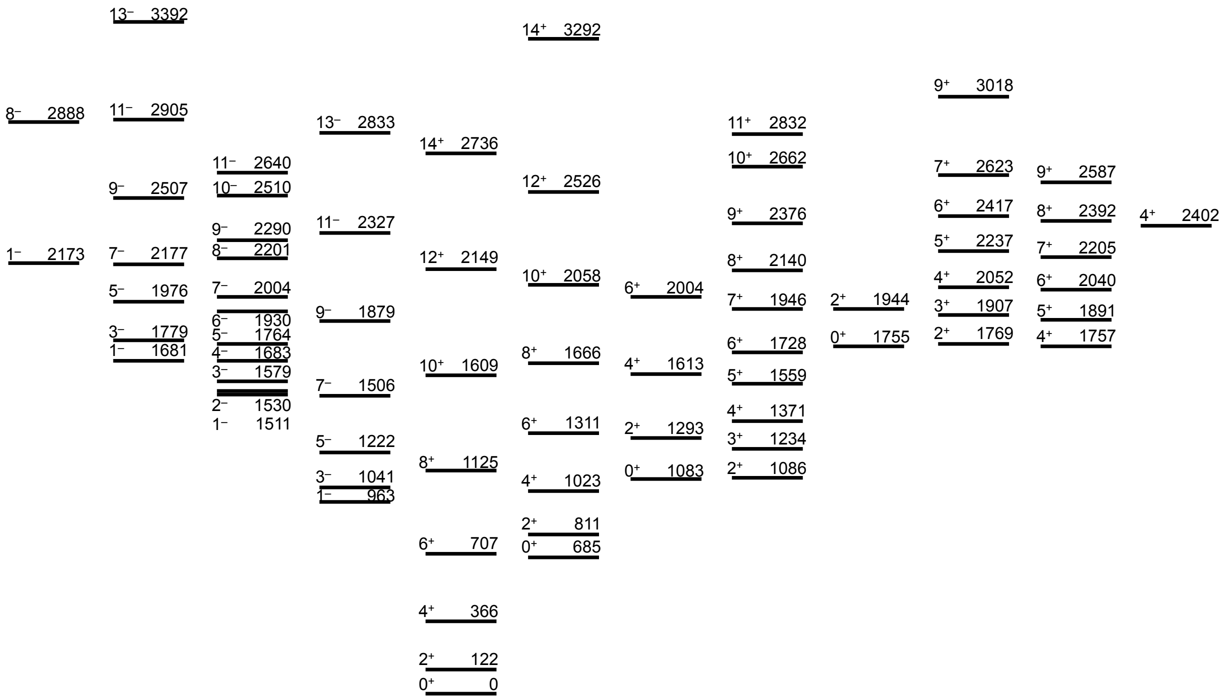


$$\langle q^2 \rangle = \sum_r \langle 0_1^+ \| \hat{Q} \| 2_r^+ \rangle \langle 2_r^+ \| \hat{Q} \| 0_1^+ \rangle$$

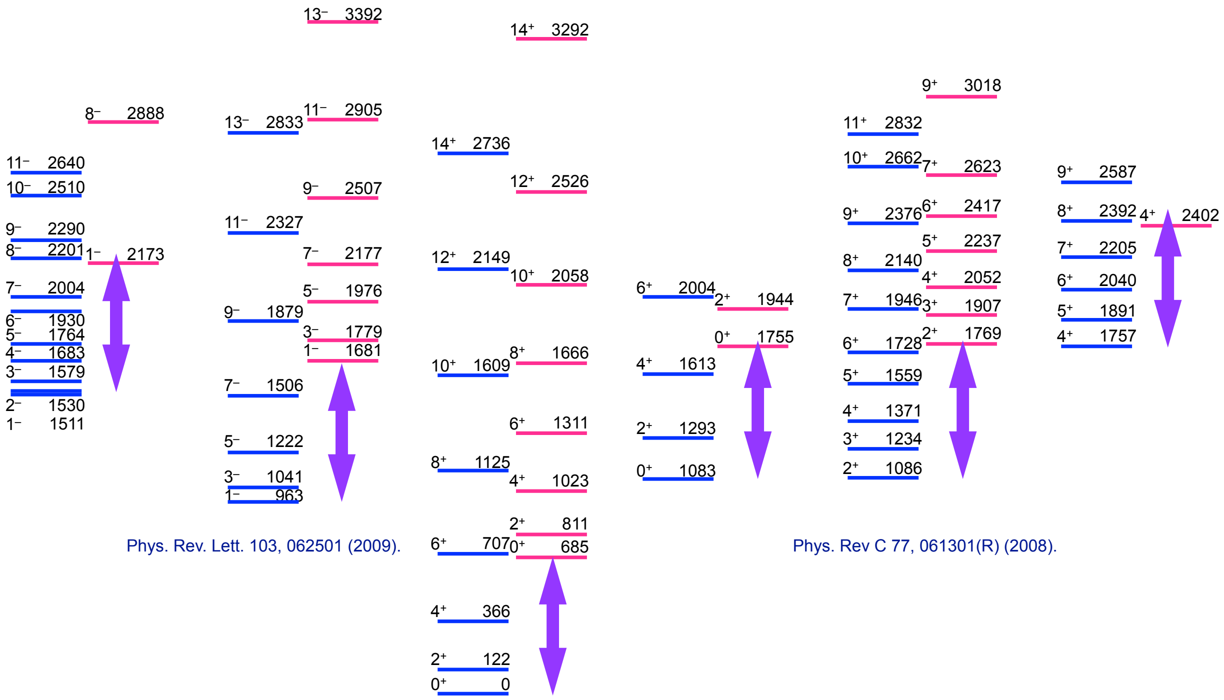
$$\langle q^3 \cos 3\delta \rangle = -\sqrt{\frac{7}{10}} \sum_{r,s} \langle 0_1^+ \| \hat{Q} \| 2_r^+ \rangle \langle 2_r^+ \| \hat{Q} \| 2_s^+ \rangle \langle 2_s^+ \| \hat{Q} \| 0_1^+ \rangle$$

$$\langle q^4 \rangle = \sum_{r,s,t} \langle \hat{Q} \rangle_{1r} \langle \hat{Q} \rangle_{rs} \langle \hat{Q} \rangle_{st} \langle \hat{Q} \rangle_{t1} \ \{\text{recoupled}\}$$

Rearranging the bands provides an insight to the underlying structure of ^{152}Sm:



Rearranging the bands provides an insight to the underlying structure of ^{152}Sm:



Phys. Rev. Lett. 103, 062501 (2009).

Phys. Rev C 77, 061301(R) (2008).

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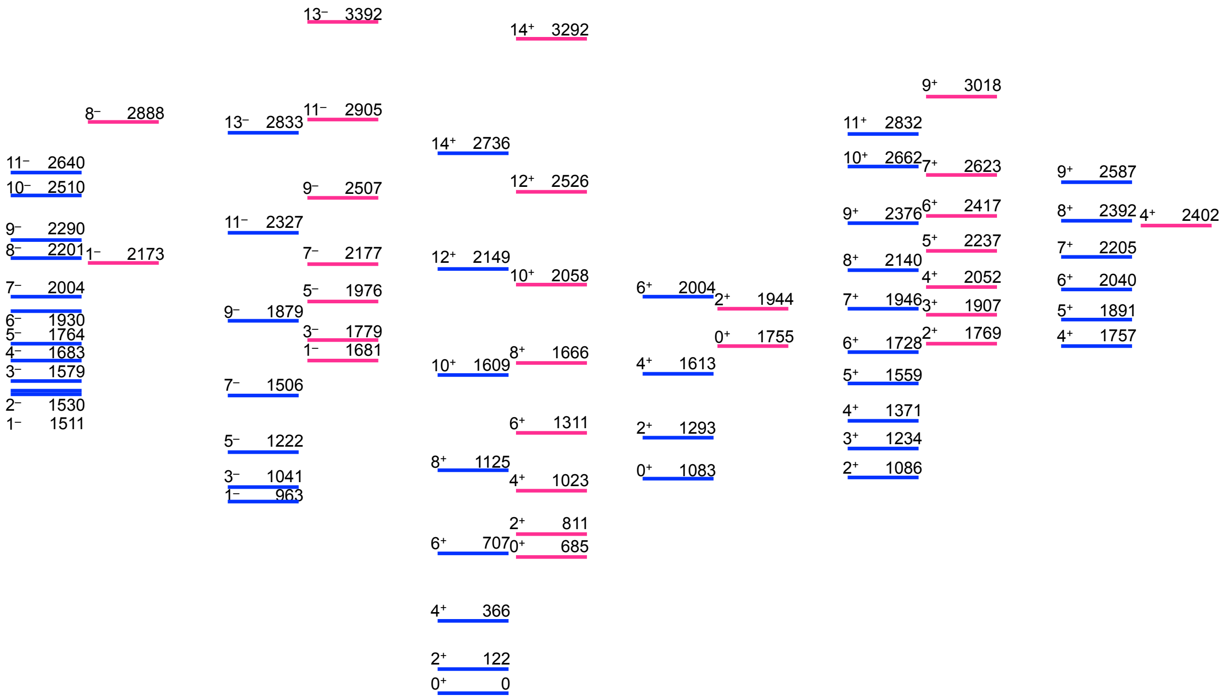

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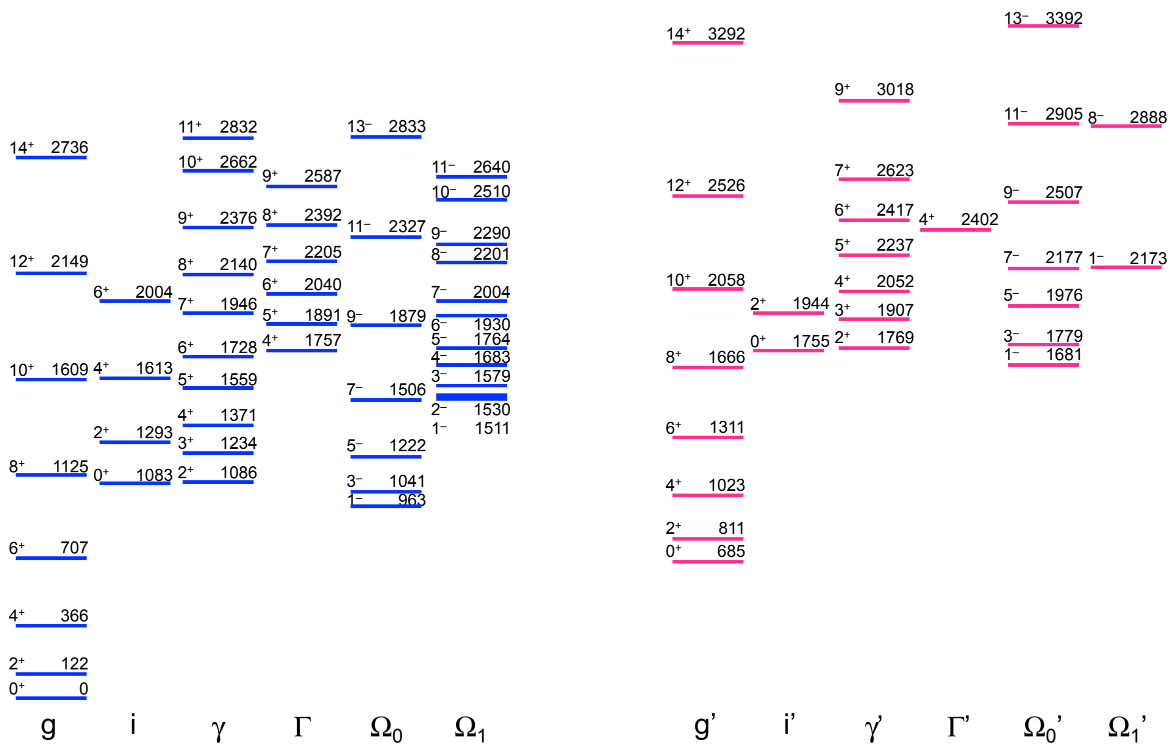
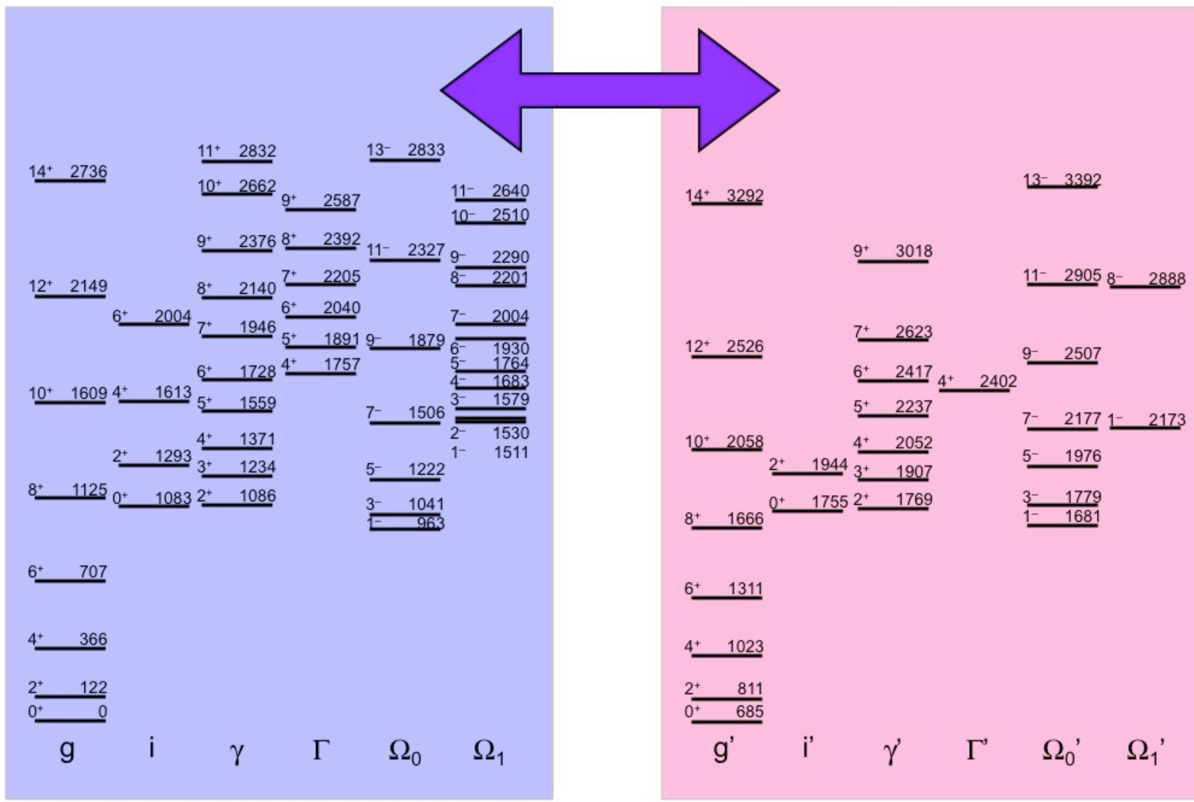
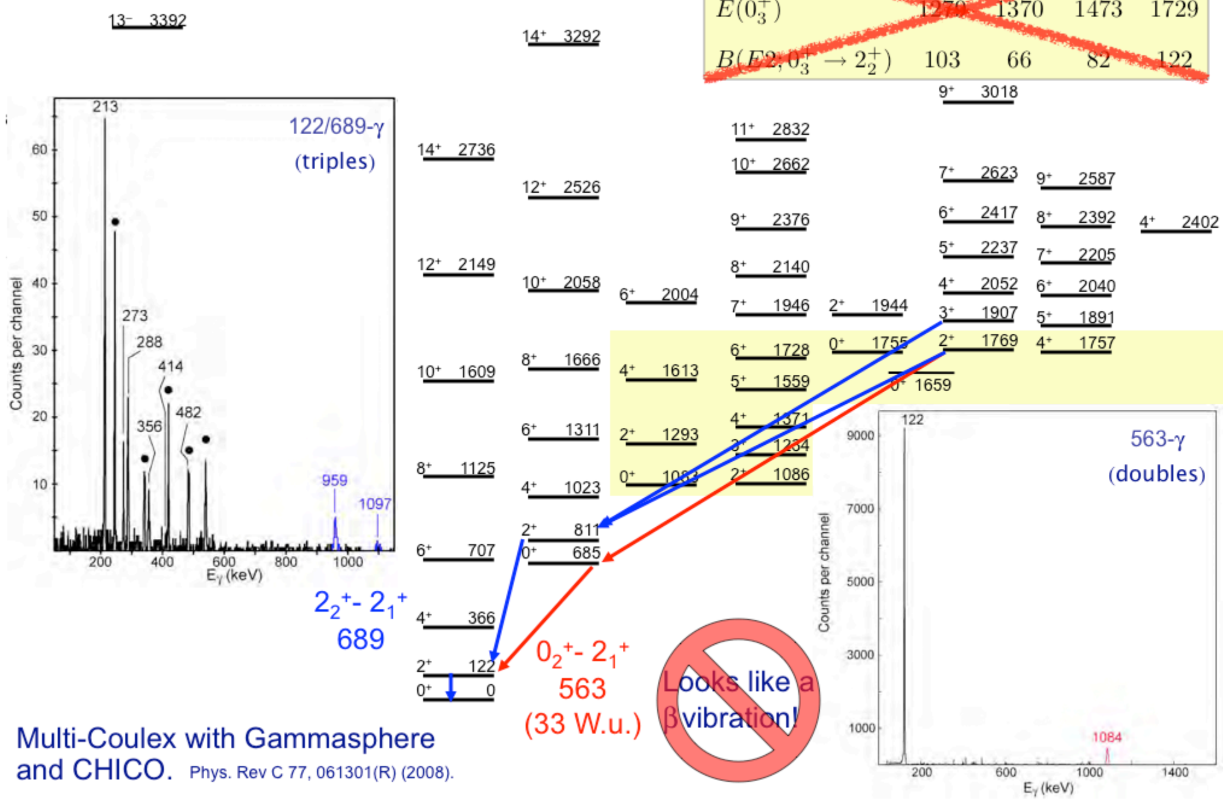


Rearranging the bands provides an insight to the underlying structure of ^{152}Sm: **a double vacuum (strongly mixed)!**



There is no evidence of a two-phonon excitation built upon the first excited 0^+ state in ^{152}Sm.

	IBM	QHV	PPQ	X(5)
$E(0_3^+)$	1270	1370	1473	1729
$B(E2; 0_3^+ \rightarrow 2_2^+)$	103	66	82	122



Multi-Coulex with Gammasphere and CHICO. Phys. Rev C 77, 061301(R) (2008).

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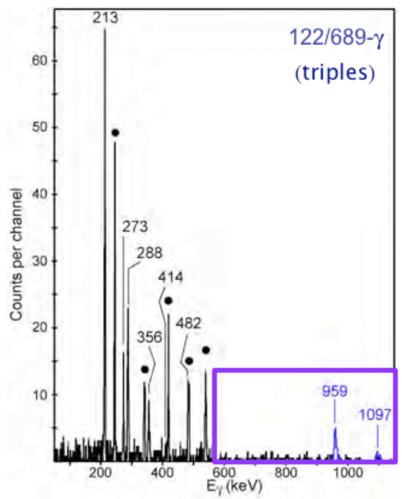
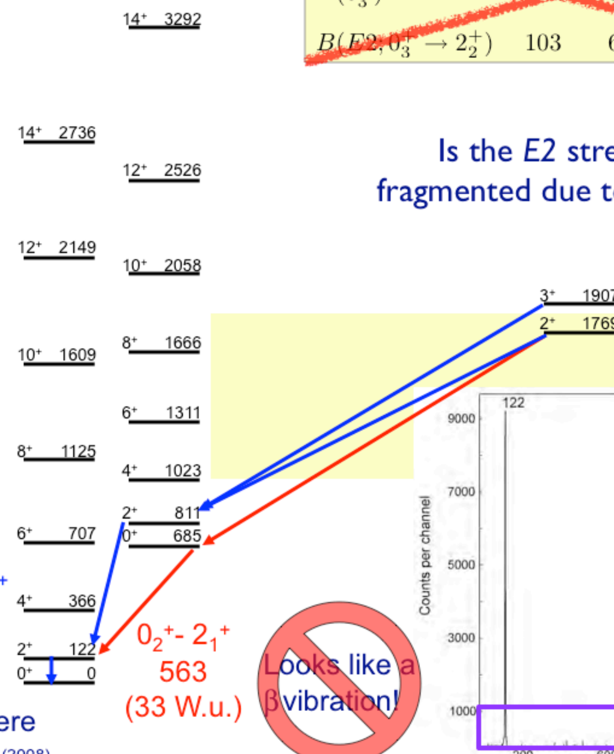
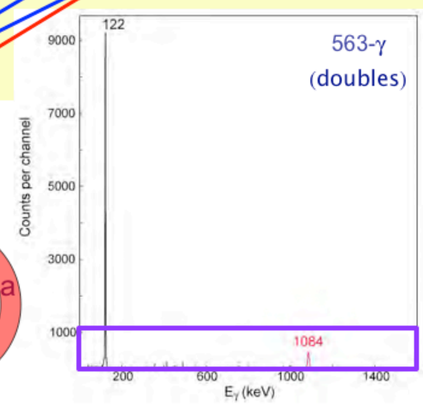
Is the *E2* strength fragmented due to mixing?







Multi-Coulex with Gammasphere and CHICO. Phys. Rev C 77, 061301(R) (2008).

There is no evidence for significant fragmented strength distributed among the excited 0^+ states populated in multi-Coulex.

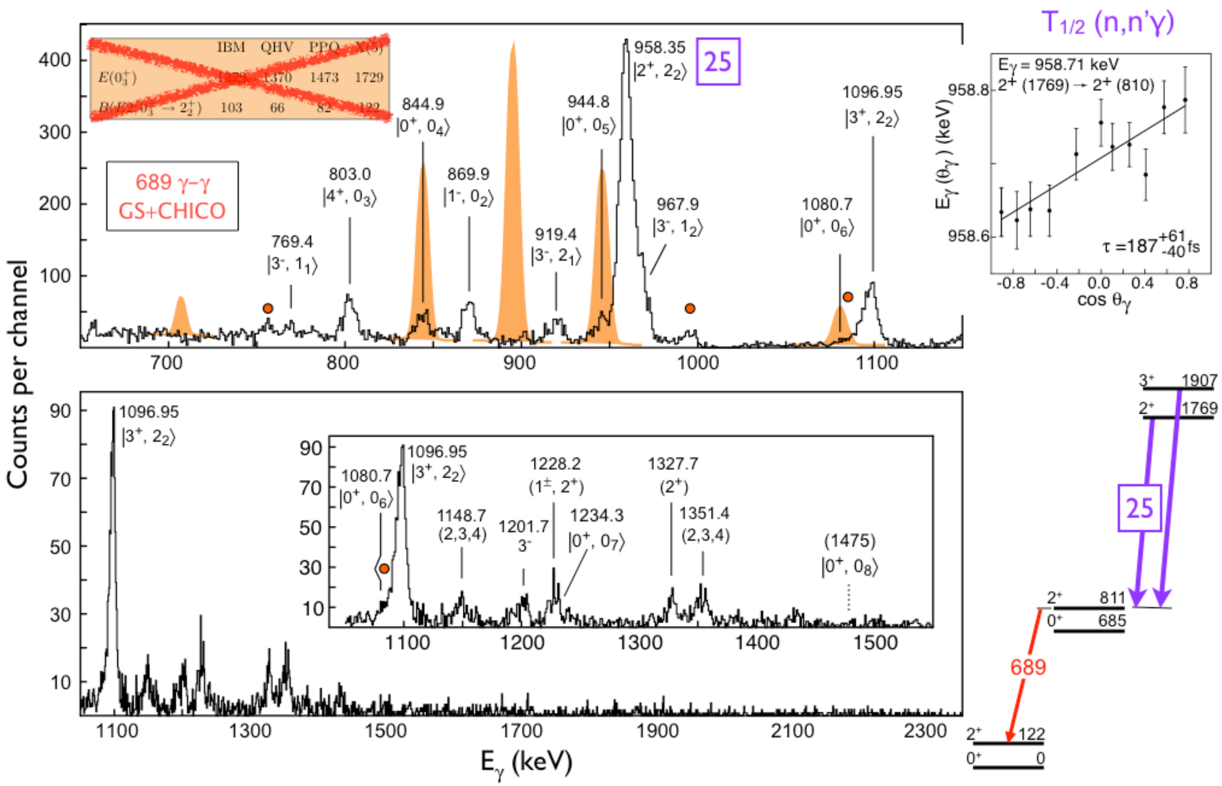


Table 2.3: Collective properties of even rare-earth isotopes as defined in the text. The model predicts that $E^+_{4_1}/E^+_{2_1}$ should be equal to 10/3, that y/x should equal $E_{2_1}/(2\hbar\omega_\beta)$, and z/x should equal $E_{2_1}/(\hbar\omega_\gamma)$. (The data are taken from Garrett P.E. (2001), *J. Phys. G: Nucl. Part. Phys.* **27**, R1 and *Nuclear Data Sheets*.)

Isotope	E_{2_1} keV	$\frac{E_{4_1}}{E_{2_1}}$	x W.u.	E_{2_β} keV	y W.u.	E_{2_γ} keV	z W.u.	$\frac{y}{x}$	$\frac{E_{2_1}}{2\hbar\omega_\beta}$	$\frac{z}{x}$	$\frac{E_{2_1}}{\hbar\omega_\gamma}$
^{154}Sm	82.0	3.25	177	1178	0.94^{23}	1440	3.2^{5}	0.005	0.037	0.018	0.057
^{156}Gd	89.0	3.24	185	1129	0.63^{6}	1154	4.69^{17}	0.003	0.043	0.025	0.078
^{158}Gd	79.5	3.29	197	1260	0.31^{4}	1187	3.4^{4}	0.002	0.033	0.018	0.066
^{160}Gd	75.3	3.30	202	1377		988	3.80^{22}		0.028	0.019	0.075
^{158}Dy	78.9	3.21	183	1086	2.1^{5}	946	5.9^{12}	0.011	0.039	0.032	0.084
^{160}Dy	86.8	3.27	198	1350	0.65^{8}	966	4.5^{3}	0.004	0.034	0.023	0.090
^{162}Dy	80.7	3.29	203	1453		888	4.59^{31}		0.030	0.023	0.090
^{164}Dy	73.4	3.30	209	1716		762	4.0^{4}			0.019	0.096
^{162}Er	102.0	3.23	190	1171	1.6^{10}	901	6.2^{3}	0.008	0.048	0.032	0.114
^{164}Er	91.4	3.28	203	1315	0.23^{12}	860	5.2^{6}	0.001	0.037	0.024	0.105
^{166}Er	80.6	3.29	214	1528		786	5.5^{4}			0.026	0.102
^{168}Er	79.8	3.31	209	1276	0.06^{1}	821	4.80^{17}	0.0003	0.033	0.023	0.096
^{170}Er	78.6	3.31	207	960	0.28^{3}	934	3.68^{11}	0.001	0.045	0.018	0.084
^{168}Yb	87.7	3.27	201	1233	1.8^{2}	984	4.60^{10}	0.009	0.039	0.022	0.090
^{170}Yb	84.3	3.29	206	1139	1.08^{21}	1146	2.7^{6}	0.005	0.040	0.013	0.075
^{172}Yb	78.7	3.31	211	1118	0.24^{1}	1466	1.33^{11}	0.001	0.038	0.0063	0.054
^{174}Yb	76.5	3.31	205	1561	0.54^{23}	1634	2.5^{5}		0.026	0.0087	0.048
^{176}Yb	82.1	3.31	180	1200		1261	1.8^{2}			0.0093	0.066
^{174}Hf	91.0	3.27	168	900	2.1^{6}	1227	4.8^{22}	0.014	0.057	0.032	0.075
^{176}Hf	88.3	3.28	179	1227	1.0^{2}	1341	3.9^{6}	0.006	0.039	0.022	0.066
^{178}Hf	93.2	3.29	161	1277	0.061^{25}	1175	3.9^{5}	0.005	0.039	0.025	0.078
^{180}Hf	93.3	3.31	154	1183		1200	3.8^{6}		0.044	0.025	0.078
^{182}W	100.1	3.29	136	1257	0.91^{8}	1221	3.40^{9}	0.007	0.044	0.025	0.081
^{184}W	111.2	3.27	121	1121	0.21^{3}	903	4.41^{22}	0.002	0.049	0.037	0.111
^{186}W	122.3	3.23	110	1015		738	4.63^{23}		0.067	0.042	0.165

R&W:
β vibrations
γ vibrations

Theory:
(adiabatic Bohr model)

$$\frac{y}{x} = \frac{E_{2_1}}{2\hbar\omega_\beta}$$

--fails by 4x to 40x

$$\frac{z}{x} = \frac{E_{2_1}}{\hbar\omega_\gamma}$$

--fails by 2x to 7x