LECTURE 1. Status of low-energy* quadrupole vibrations in spherical nuclei

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P.E. Garrett and J.L. Wood, J. Phys. G: Nucl. Part. Phys. 37 064028 2010

P.E. Garrett et al., Phys. Rev. C 86 044304 2012

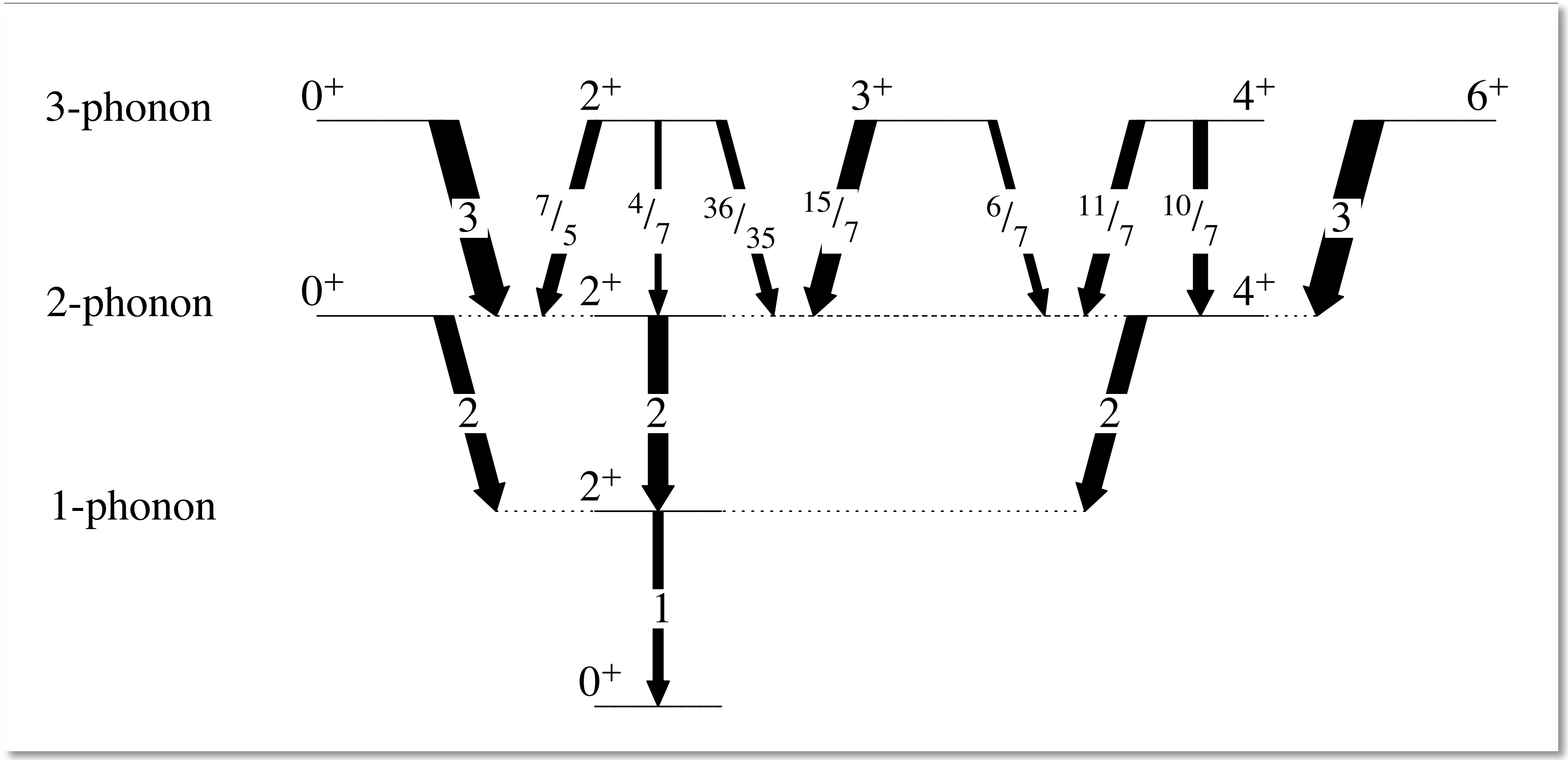
J.C. Batchelder et al., Phys. Rev. C 80 054318 2009

P.E. Garrett et al., Phys. Rev. C 78 044307 2008 [theory—IBM-2 / IBM-MIX]

*High-energy quadrupole vibrations are called giant quadrupole resonances (GQR)

Harmonic quadrupole vibrational collectivity: patterns of E_x and B(E2)

$B(E2; 2_1^+ \rightarrow 0_1^+) = 1$



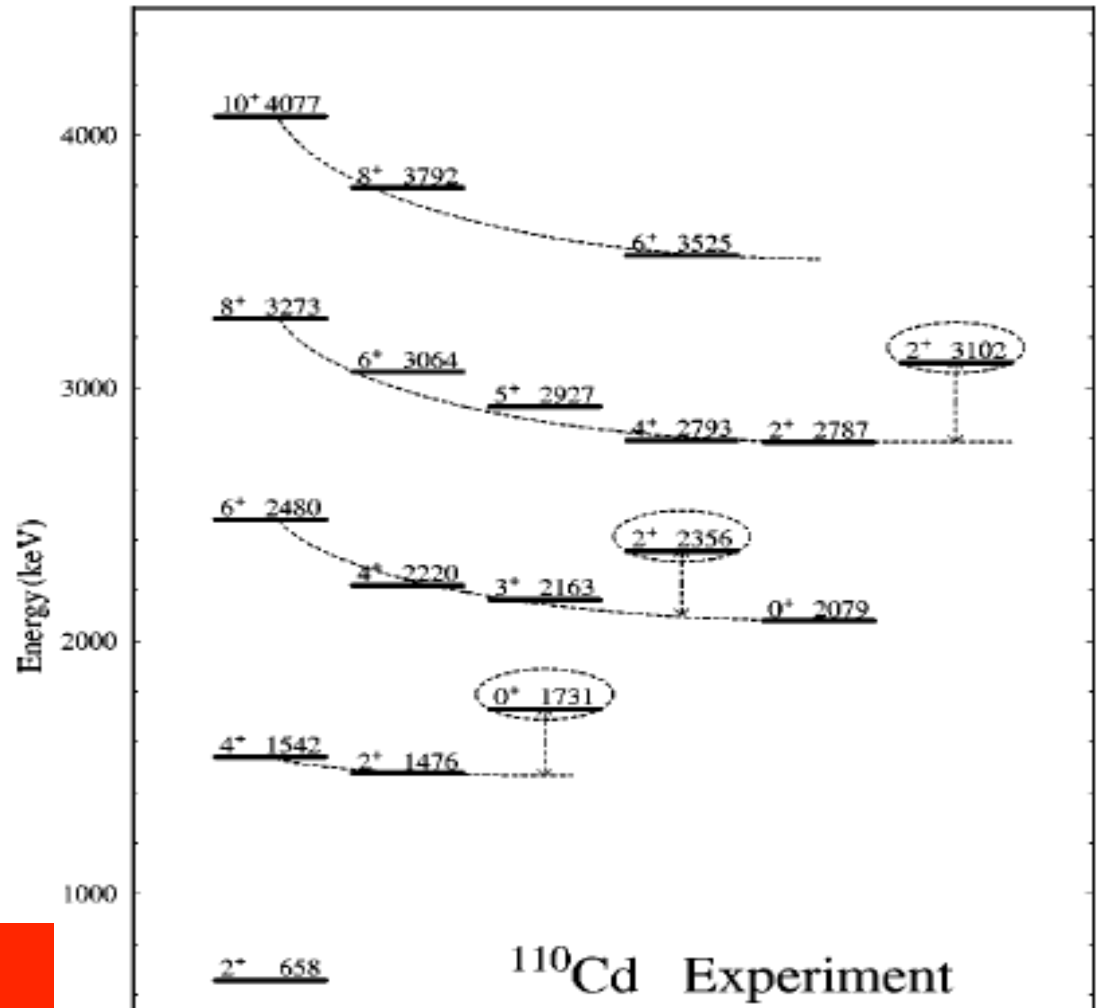
Spherical vibrational nuclei

Vibrator (H.O.)

$$E(I) = n\,(\hbar\,\omega_0)$$

$$R_{4/2} = 2.0$$

A long-promoted view of ^{110}Cd, based on energies (figure by R.F. Casten)



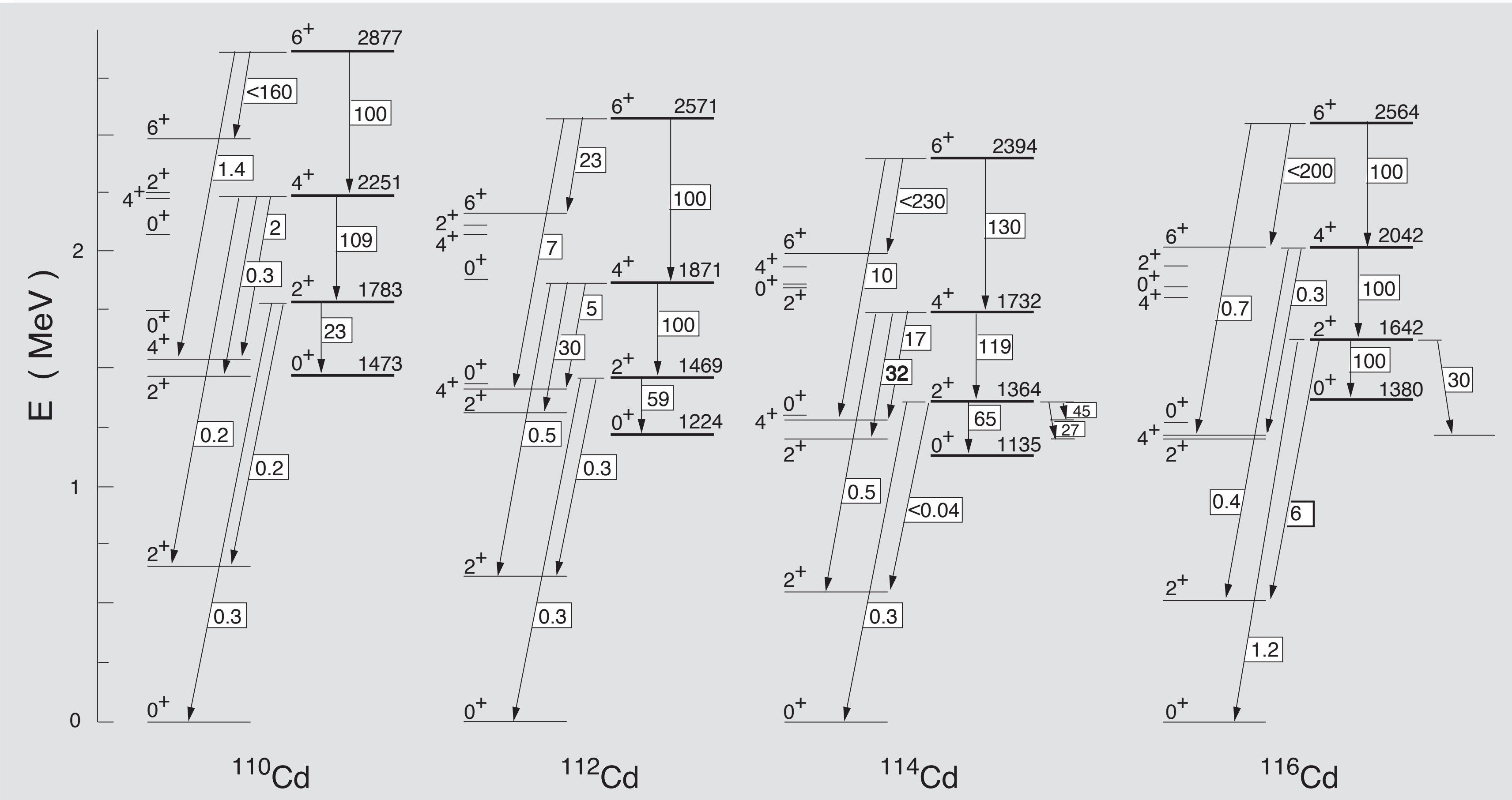
n **= 0,1,2,3,4,5 !!**

n **= phonon No.**

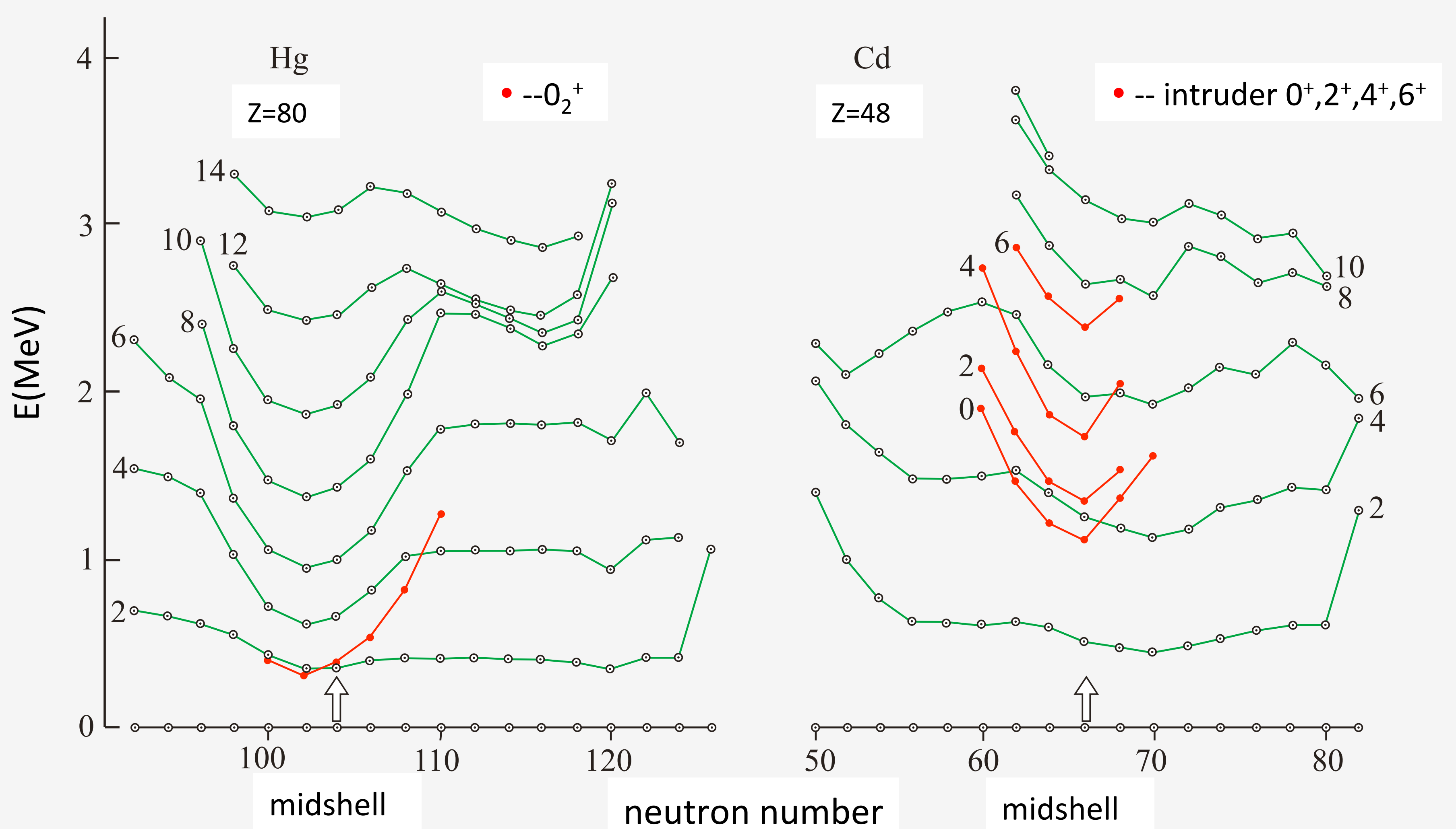
Complication: there is shape coexistence in $^{110-116}$Cd

Figure from Rowe & Wood

B(E2)'s in W.u. [100 = rel. value]

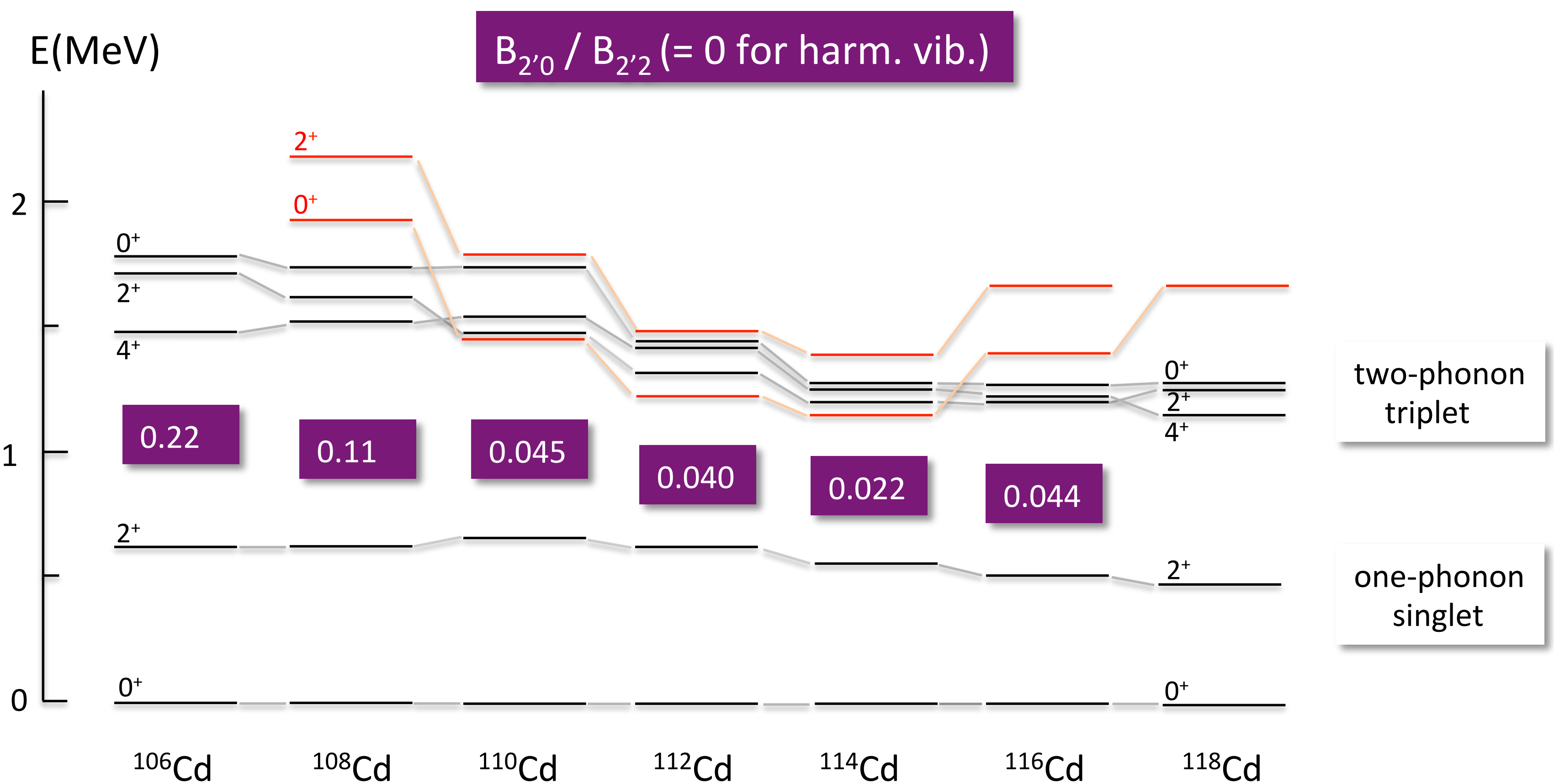


Shape coexistence in the Hg and Cd isotopes



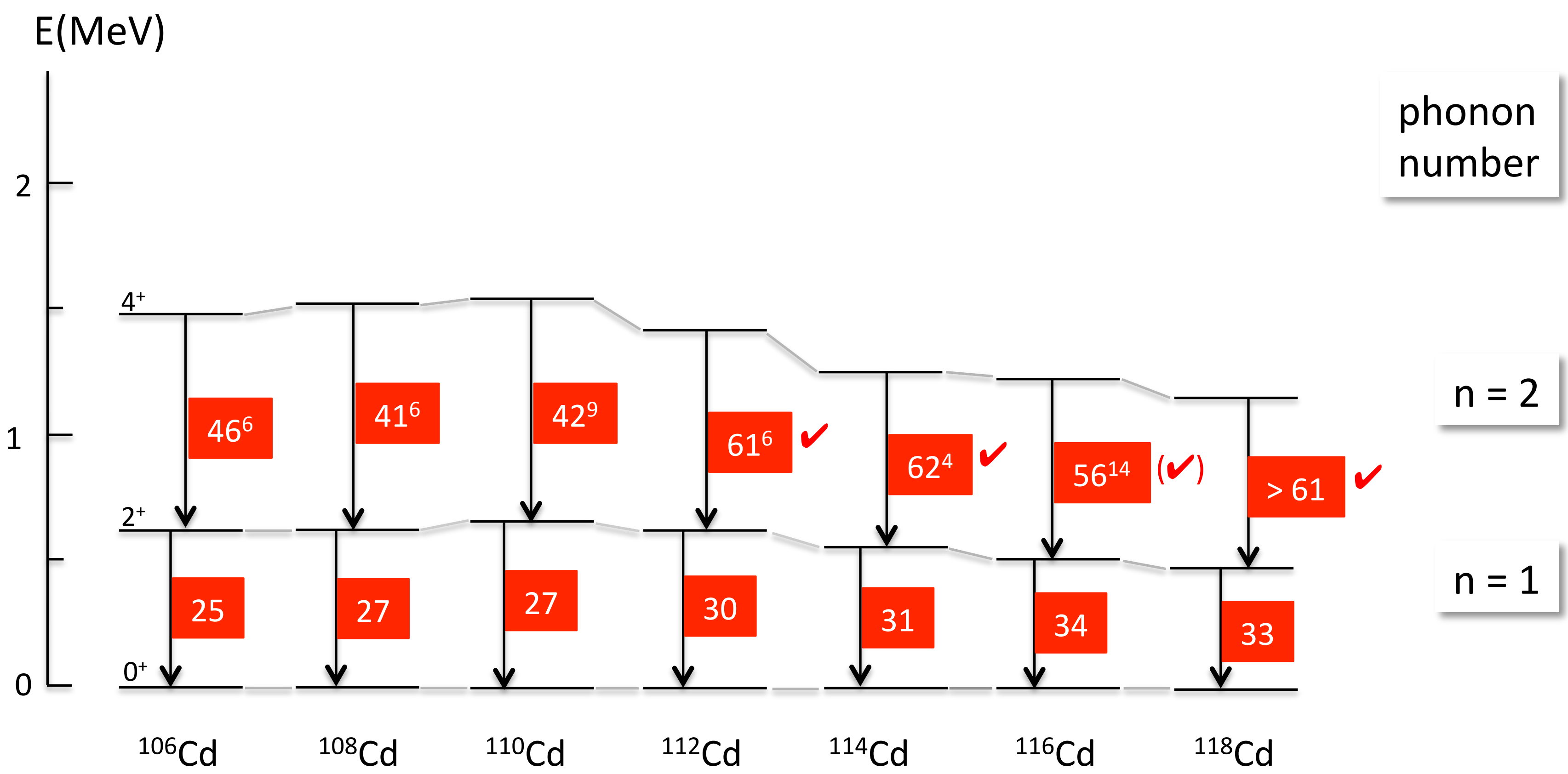
The case for quadrupole vibrational collectivity in the Cd isotopes

Prior to ~2003: “we all thought that the Cd isotopes were vibrational” –
based on energies and relative B(E2)’s…



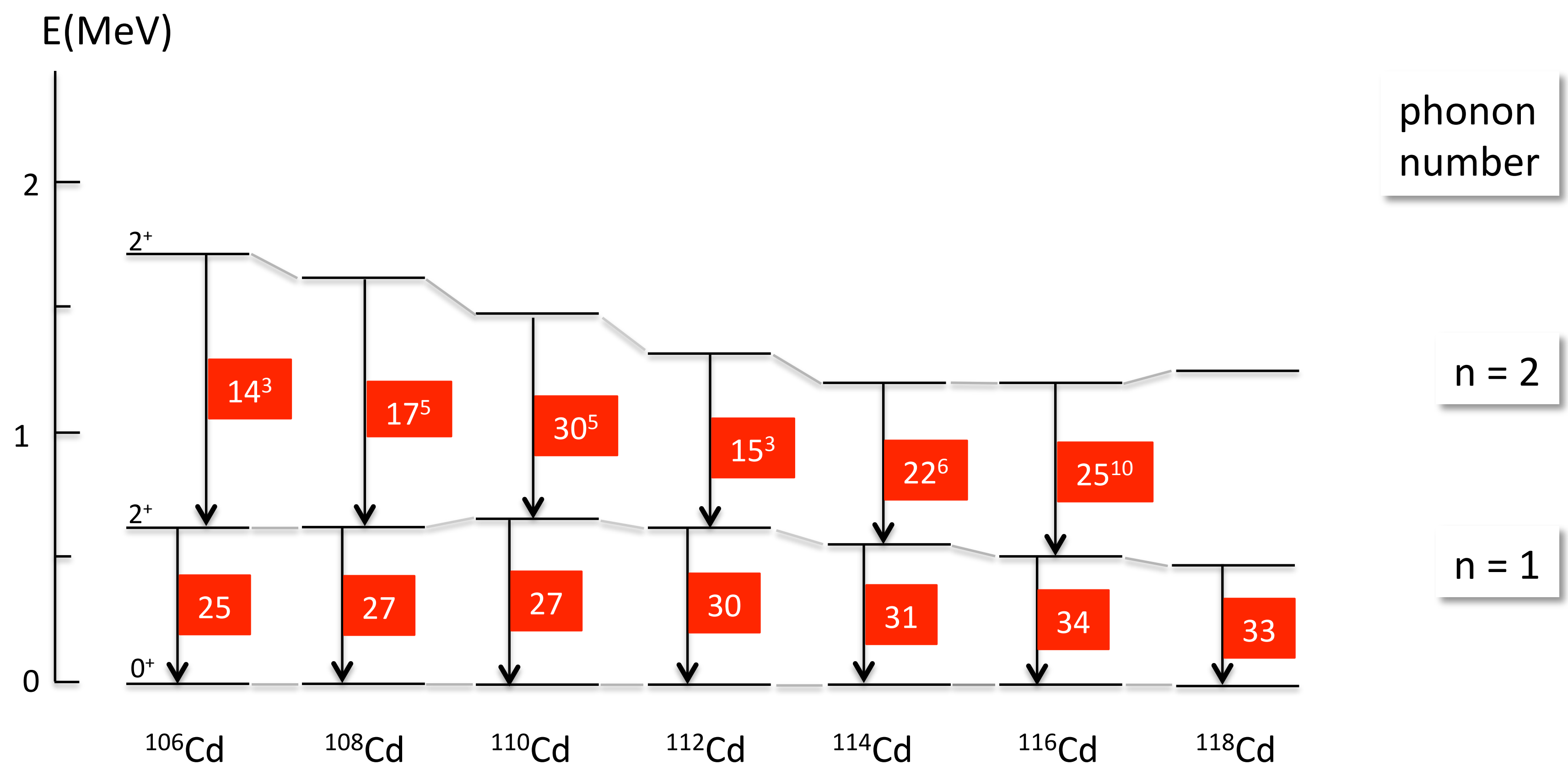
The case for quadrupole vibrational collectivity in the Cd isotopes

Prior to ~2003, we all thought that the Cd isotopes were vibrational –
based on some $B(E2; 4_1^+ \rightarrow 2_1^+)$'s …



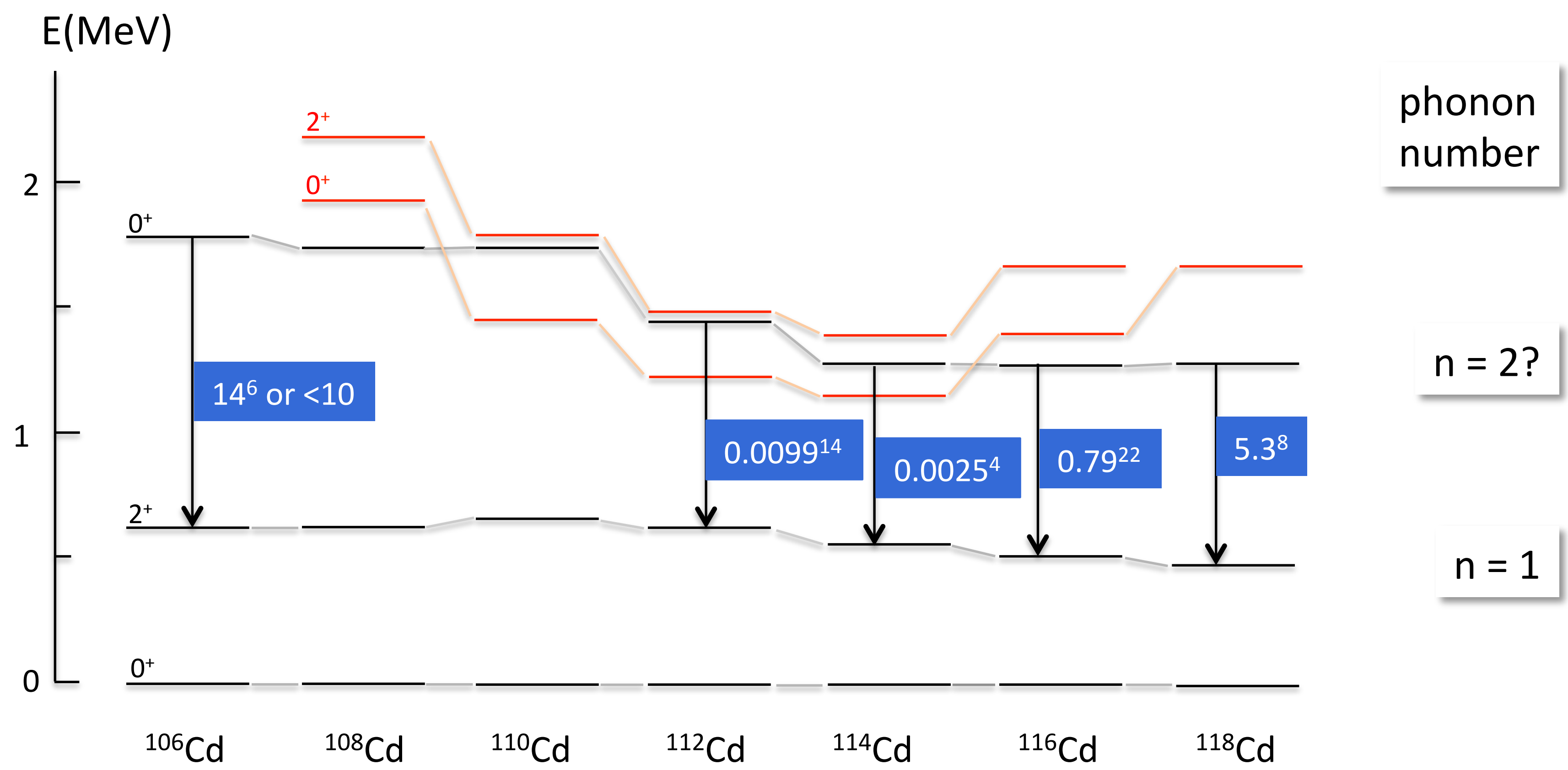
The case for quadrupole vibrational collectivity in the Cd isotopes

Prior to ~2003, we all thought that the Cd isotopes were vibrational –
but $B(E2; 2_2^+ \rightarrow 2_1^+)$'s are not strongly supportive…



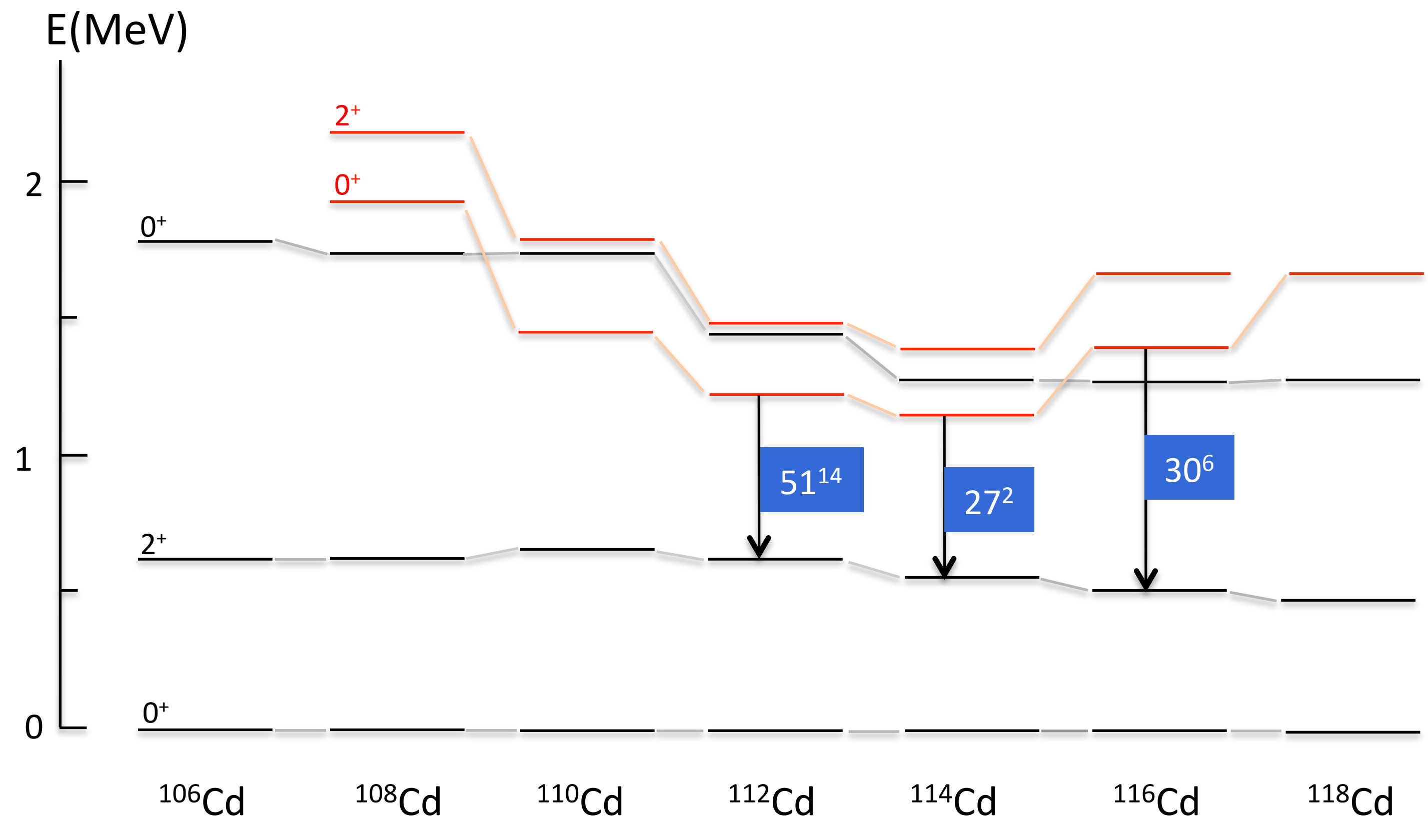
The case for quadrupole vibrational collectivity in the Cd isotopes

Prior to ~2003, we all thought that the Cd isotopes were vibrational –
and $B(E2; 0_{2,3}^+ \rightarrow 2_1^+)$'s refute vibrations; except maybe….



The case for quadrupole vibrational collectivity in the Cd isotopes

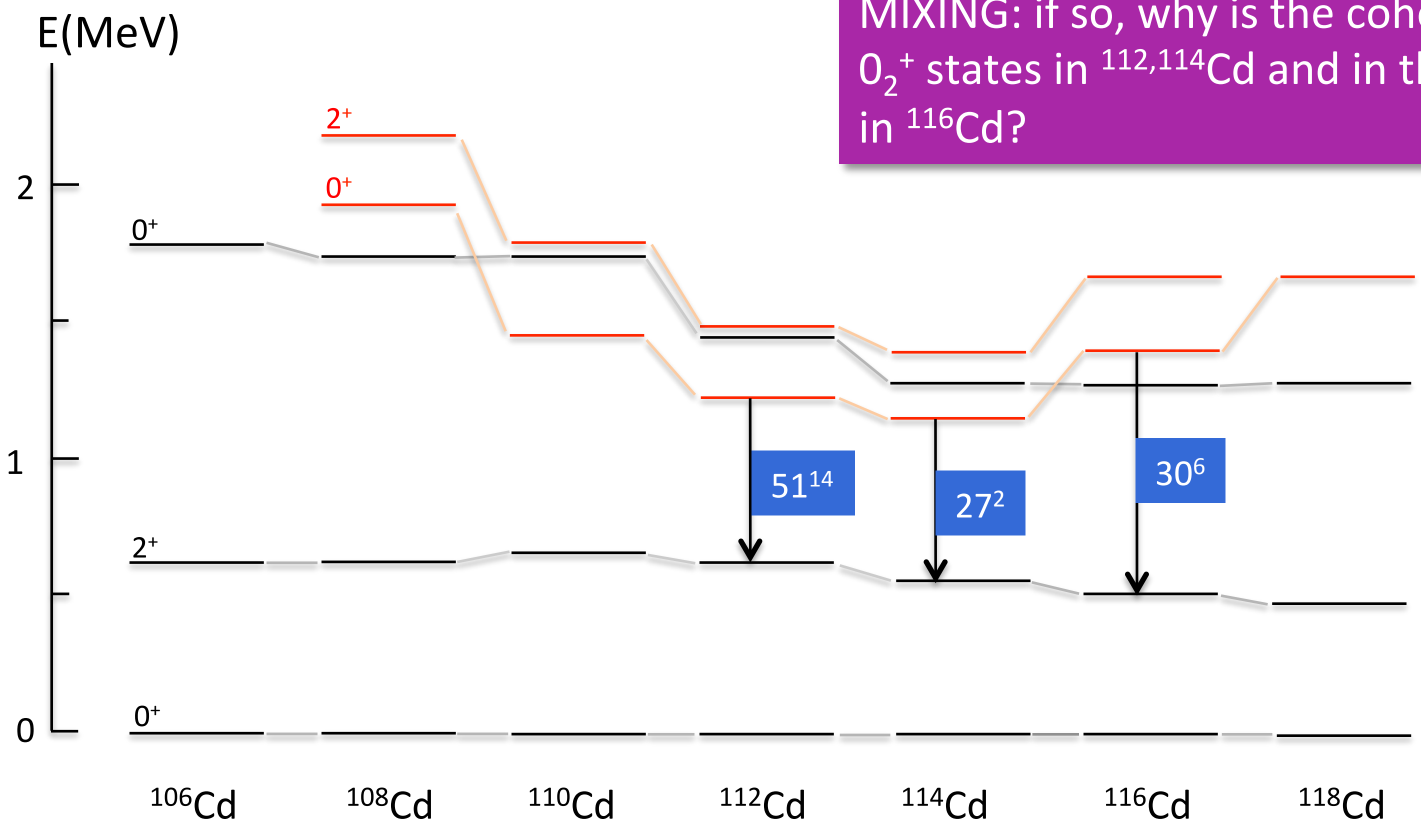
Prior to ~2003, we all thought that the Cd isotopes were vibrational – some $B(E2; 0_{3,2}^{+} \rightarrow 2_1^{+})$'s have some strength. But they appear to be from the Intruder band heads….? So, maybe there is MIXING.



The case for quadrupole vibrational collectivity in the Cd isotopes

Prior to ~2003, we all thought that the Cd isotopes were vibrational –
MIXING….

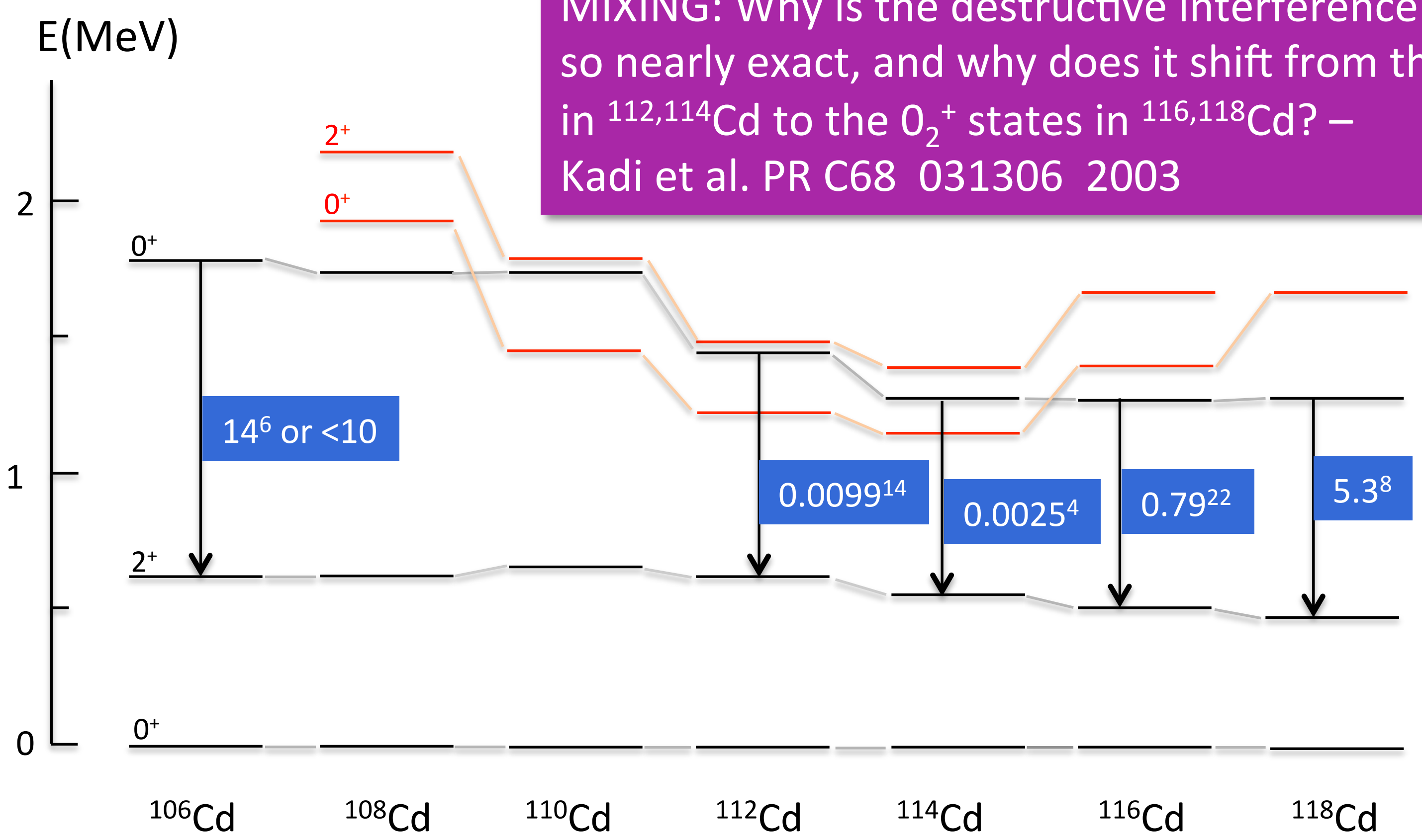
MIXING: if so, why is the coherence in the 0_2^+ states in $^{112,114}Cd$ and in the 0_3^+ state in ^{116}Cd?



The case for quadrupole vibrational collectivity in the Cd isotopes

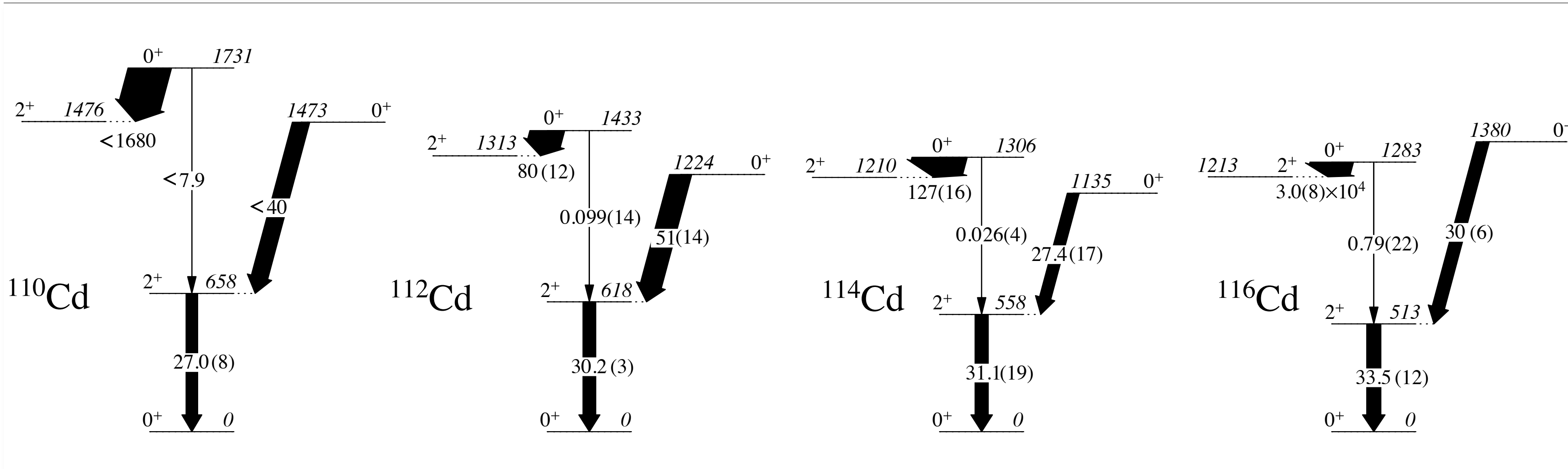
Prior to ~2003, we all thought that the Cd isotopes were vibrational –
MIXING…

MIXING: Why is the destructive interference persistently so nearly exact, and why does it shift from the 0_3^+ states in ^{112,114}Cd to the 0_2^+ states in ^{116,118}Cd? –
Kadi et al. PR C68 031306 2003



Excited 0⁺ decays in the Cd isotopes

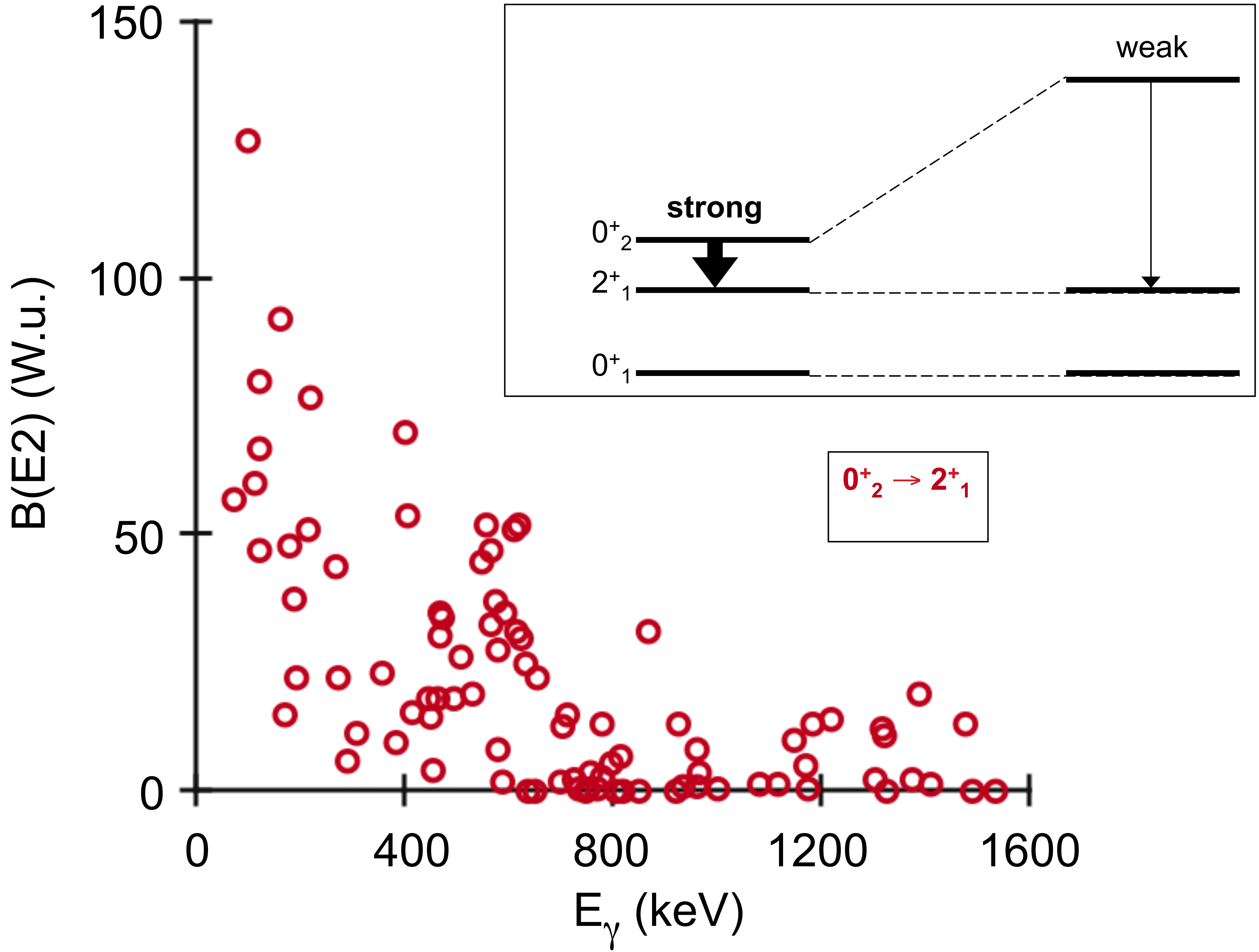
Figure taken from P.E. Garrett et al., PR C86 044304 2012



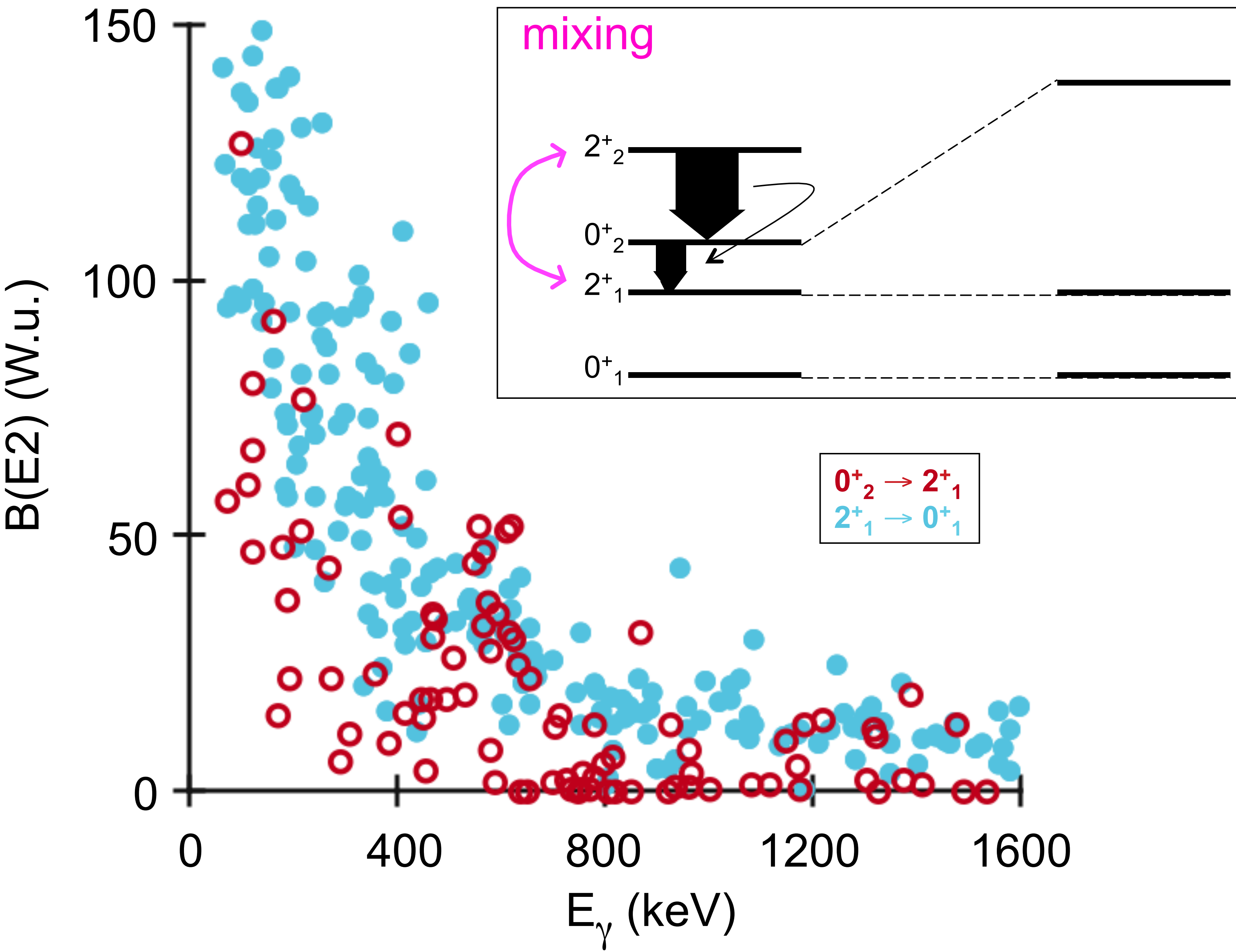
Deformed band head 0^+ states: strong E2 decay to "one-phonon" 2^+ states

"Two-phonon" 0^+ states: very weak E2 decay to "one-phonon" 2^+ states; but strong E2 decay to "two-phonon" 2^+ states

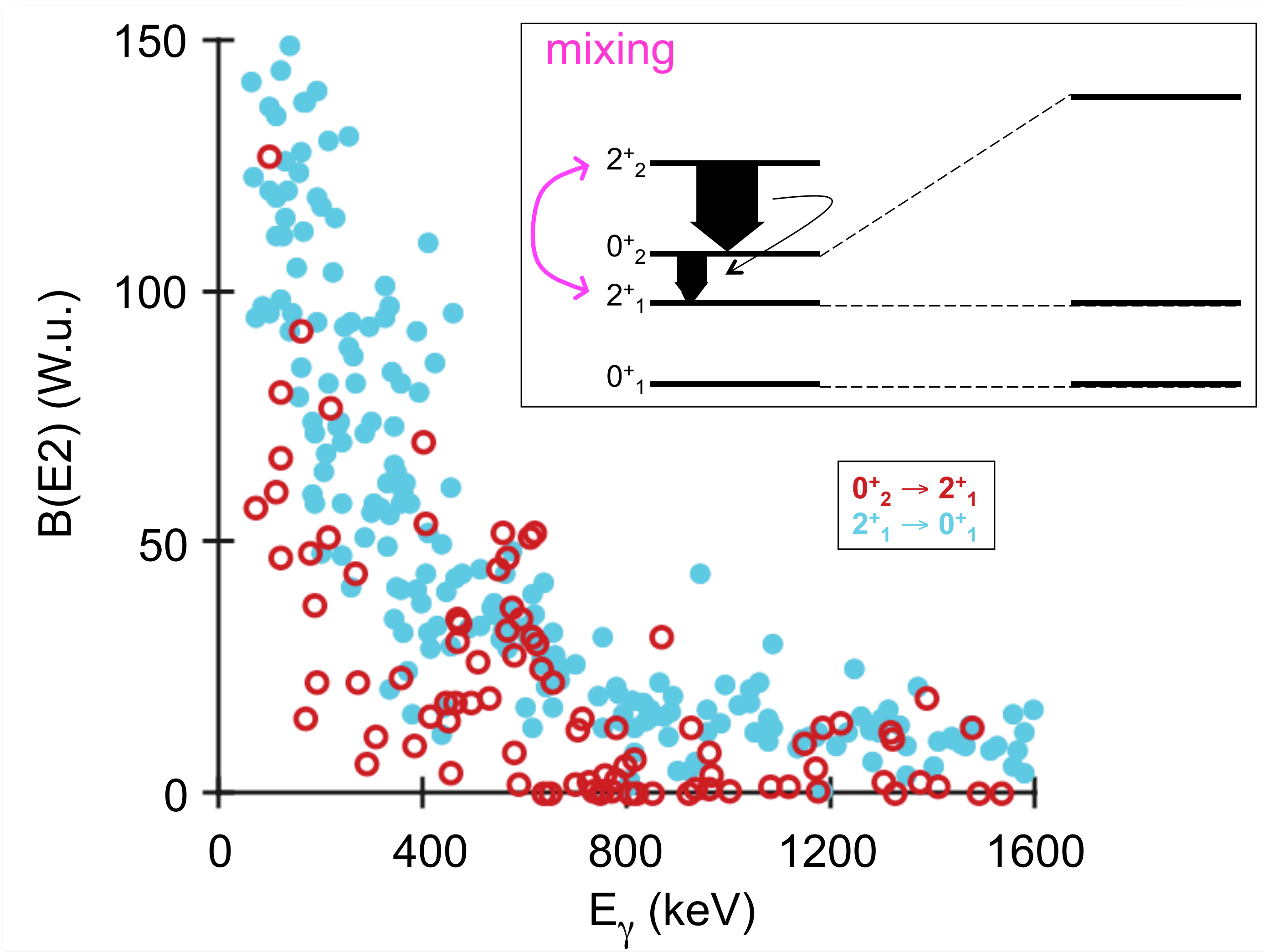
Systematics of B(E2; $0^+_2 \rightarrow 2^+_1$) vs. E_γ ($0^+_2 - 2^+_1$)



Systematics of $B(E2; 0^+_2 \rightarrow 2^+_1)$ vs. E_γ $(0^+_2 - 2^+_1)$



B(E2; $0_2^+ \rightarrow 2_1^+$) vs. E(0_2^+) – E(2_1^+): coexistence and mixing yields B(E2; $0_2^+ \rightarrow 2_1^+$) ~ α^2 β^2 $(\Delta Q)^2$



Shape coexistence, E0 transitions, and mixing

E0 transition strengths are a measure of the off-diagonal matrix elements of the mean-square charge radius operator.

Mixing of configurations with different mean-square charge radii produces E0 transition strength.

$$\rho^2(E0) = \frac{1}{\Omega\,\tau(E0)}$$

"Electronic factor"

$$\Omega = \Omega(Z, \Delta E) = \Omega_K + \Omega_{L_1} + \cdots + \Omega_{e^+e^-}$$

e.g., $Z = 80$, $\Delta E = 500\,\mathrm{keV}$, $\rho^2 = 1\times10^{-3} \Rightarrow \tau \simeq 3\,\mathrm{ns}$

Monopole strength parameter

$$\rho_{if}(E0) = \frac{\langle f|\sum_j e_j r_j^2|i\rangle}{eR^2} \equiv \frac{\langle f|m(E0)|i\rangle}{eR^2} \equiv \frac{M_{if}(E0)}{eR^2}$$

Sum is over nucleons; e_j is the effective monopole charge; $R = 1.2\,A^{1/3}\,\mathrm{fm}$; e is the unit of elec. charge.

$$|i\rangle = \alpha|1\rangle + \beta|2\rangle, \quad |f\rangle = -\beta|1\rangle + \alpha|2\rangle$$

$$M_{if}(E0) = \alpha\beta\{\langle 2|m(E0)|2\rangle - \langle 1|m(E0)|1\rangle\} + (\alpha^2-\beta^2)\langle 1|m(E0)|2\rangle$$

$$\boxed{M_{if}(E0) \simeq \alpha\beta\,\Delta\langle r^2\rangle}$$

$$\Delta\langle r^2\rangle \equiv -\langle 1|\sum_j e_j r_j^2|1\rangle + \langle 2|\sum_j e_j r_j^2|2\rangle$$

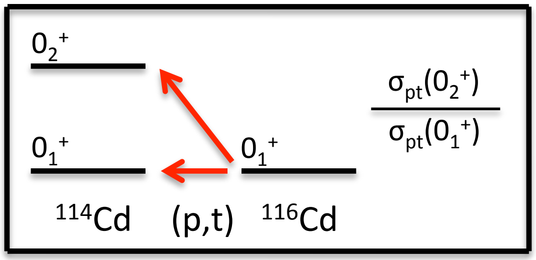
Ω values: http://bricc.anu.edu.au

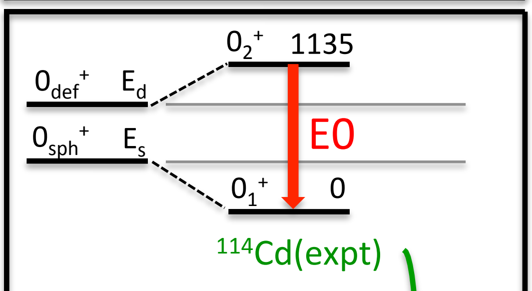
τ: partial lifetime for E0 decay branch

J. Kantele et al. Z. Phys. A289 157 1979
and see
JLW et al. Nucl. Phys. A651 323 1999

Spectroscopy of mixing in the Cd isotopes: ^{116}Cd (p,t) ^{114}Cd and ρ^2 (E0) $\cdot$ 10^3

•





Fortune PR C35 2318 1987

$|V_{J=0}|$ ~ 330 keV

β_0 / α_0 ~ 0.28 $\alpha_0^2 + \beta_0^2 = 1$

$\rho_{0\to 0}^2$ (E0) ~ $[\Delta\langle r^2\rangle]^2 \alpha_0^2 \beta_0^2$

$\Delta\langle r^2\rangle = \langle r^2\rangle_{def} - \langle r^2\rangle_{sph}$ [unknown]

$$\rho_{0\to 0}^2 (E0) \cdot 10^3 = 19 = \frac{48^2 [\Delta\langle r^2\rangle]^2 \ 10^3 \ [0.28 \times 0.96]^2}{[1.2 \times 114^{1/3}]^4}$$

$\Delta\langle r^2\rangle$ ~ 0.4 fm^2 [first estimate]

$\rho_{J\to J}^2$ (E0) $\cdot$ 10^3 ~ 300 $\alpha_J^2 \beta_J^2$

Wood et al.
NP A651 323 1999

Spectroscopy of mixing in the Cd isotopes: ρ^2 (E0)$\cdot 10^3$ values in ^{114}Cd

$\rho^2(E0) \cdot 10^3$ max ~ 100 (obs.)

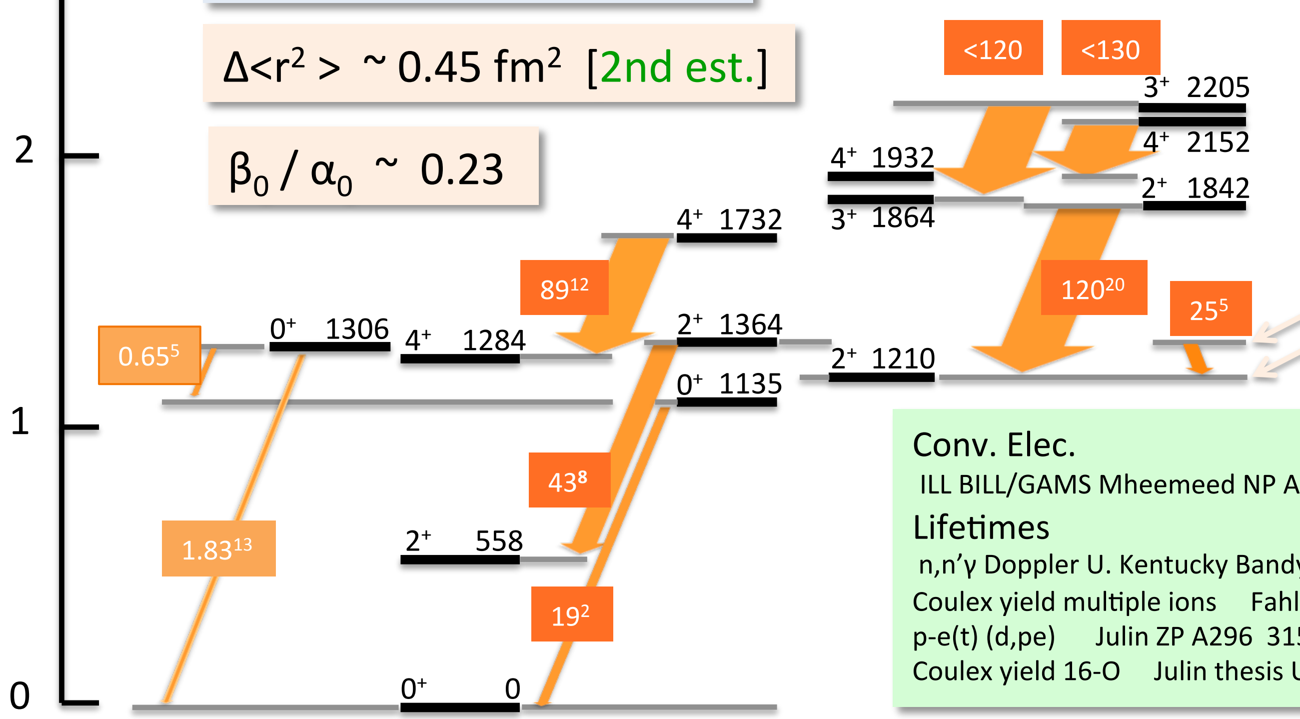
$\rho_{J \rightarrow J}^2 (E0) \cdot 10^3 \sim 400\, \alpha_J^2\, \beta_J^2$

$\Delta \langle r^2 \rangle \sim 0.45$ fm^2 [2nd est.]

$\beta_0 / \alpha_0 \sim 0.23$

$\rho^2(E0) \cdot 10^3$

^{114}Cd



NOTE:
despite the proximity of these two 2^+ states, the ρ^2 (E0) value is not large

Conv. Elec.

ILL BILL/GAMS Mheemeed NP A412 113 1984

Lifetimes

n,n'γ Doppler U. Kentucky Bandyopadhyay PR C76 054308 2007

Coulex yield multiple ions Fahlander NP A485 327 1988

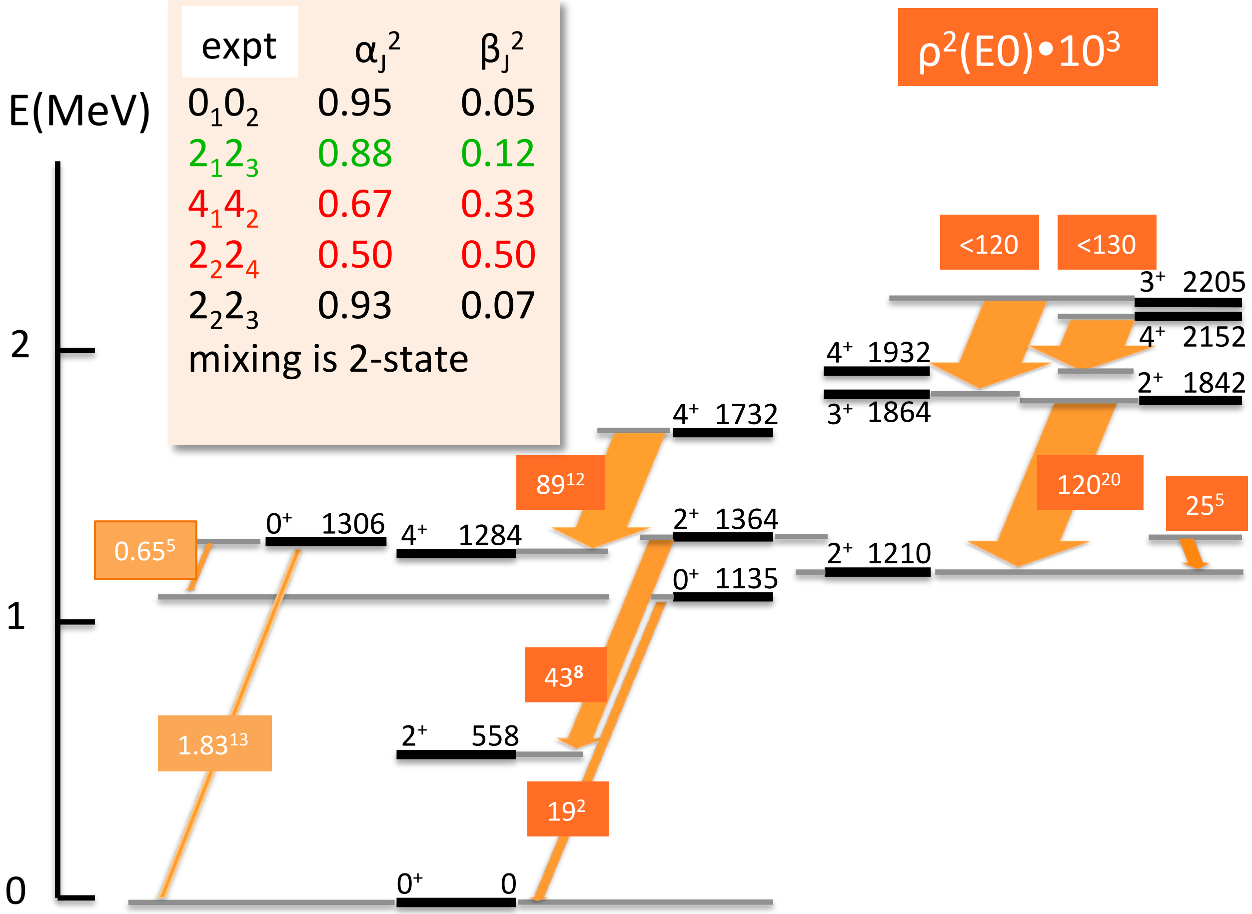
p-e(t) (d,pe) Julin ZP A296 315 1980

Coulex yield 16-O Julin thesis Univ. Jyvaskyla 1979

Spectroscopy of mixing in the Cd isotopes: ρ^2 (E0)$\cdot 10^3$ values in ^{114}Cd

expt	α_J^2	β_J^2
$0_1 0_2$	0.95	0.05
$2_1 2_3$	0.88	0.12
$4_1 4_2$	0.67	0.33
$2_2 2_4$	0.50	0.50
$2_2 2_3$	0.93	0.07

mixing is 2-state



^{114}Cd

th	α_J^2	β_J^2
$0_1 0_2$	0.99	0.01
$2_1 2_3$	0.97	0.02
$4_1 4_2$	0.94	0.05
$2_2 2_4$	0.87	0.04
$2_2 2_3$	0.87	0.09

Garrett, Green, Wood
PR C78 044307 2008
To fit B(E2)'s

IBM2-MIX
mixing is multi-state

Quadrupole vibrations at low energy in spherical nuclei—where are they?

- The Cd isotopes were considered to be the best examples of low-energy quadrupole vibrations in spherical nuclei.
- With the failure of such an interpretation, a serious dilemma arises:

  a). Should one decide that such excitations do not exist?

  (The best cases have been refuted.)

  b). Should one continue to search for such excitations by more detailed study of the most promising cases?

  (None of these cases are very well-characterized. The spectroscopic work needed is highly demanding.)

Higher-phonon states

- Low-energy quadrupole vibrational structure will possess higher-phonon states:

  a). What is the evidence?

  b). Failure to observe clear cases is sometimes attributed to

  mixing and fragmentation of E2 strength.

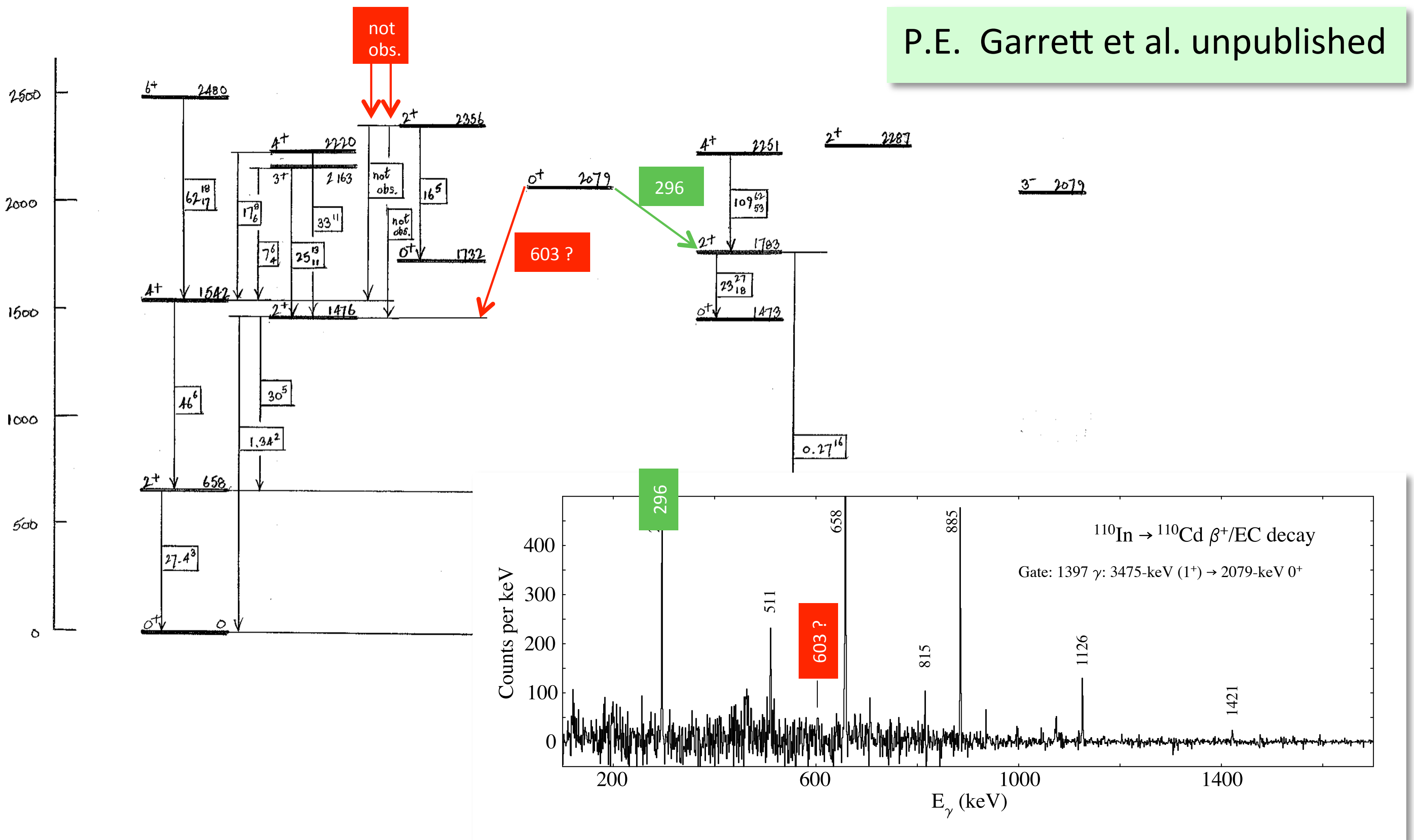
  c). How to experimentally test for fragmentation of strength:

  (i). Lifetime and branching ratio measurements.

  (ii). Multistep-Coulomb excitation measurements.

^{110}Cd: expected 3-phonon → 2-phonon transitions not all observed

P.E. Garrett et al. unpublished



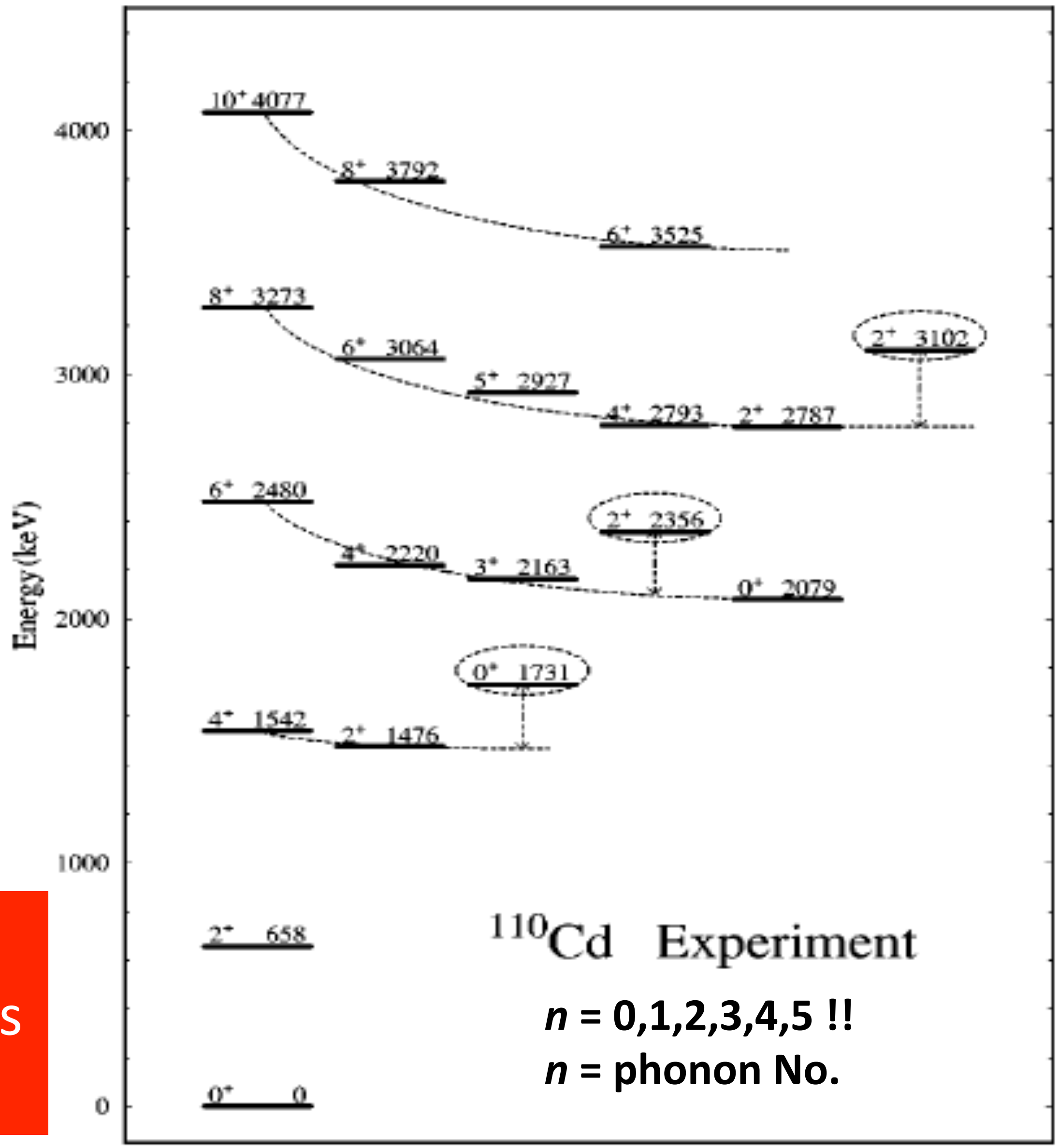
Spherical vibrational nuclei

Vibrator (H.O.)

$$E(I) = n\,(\hbar\,\omega_0)$$

$$R_{4/2} = 2.0$$

A long-promoted view of ^{110}Cd, based on energies (figure by R.F. Casten)

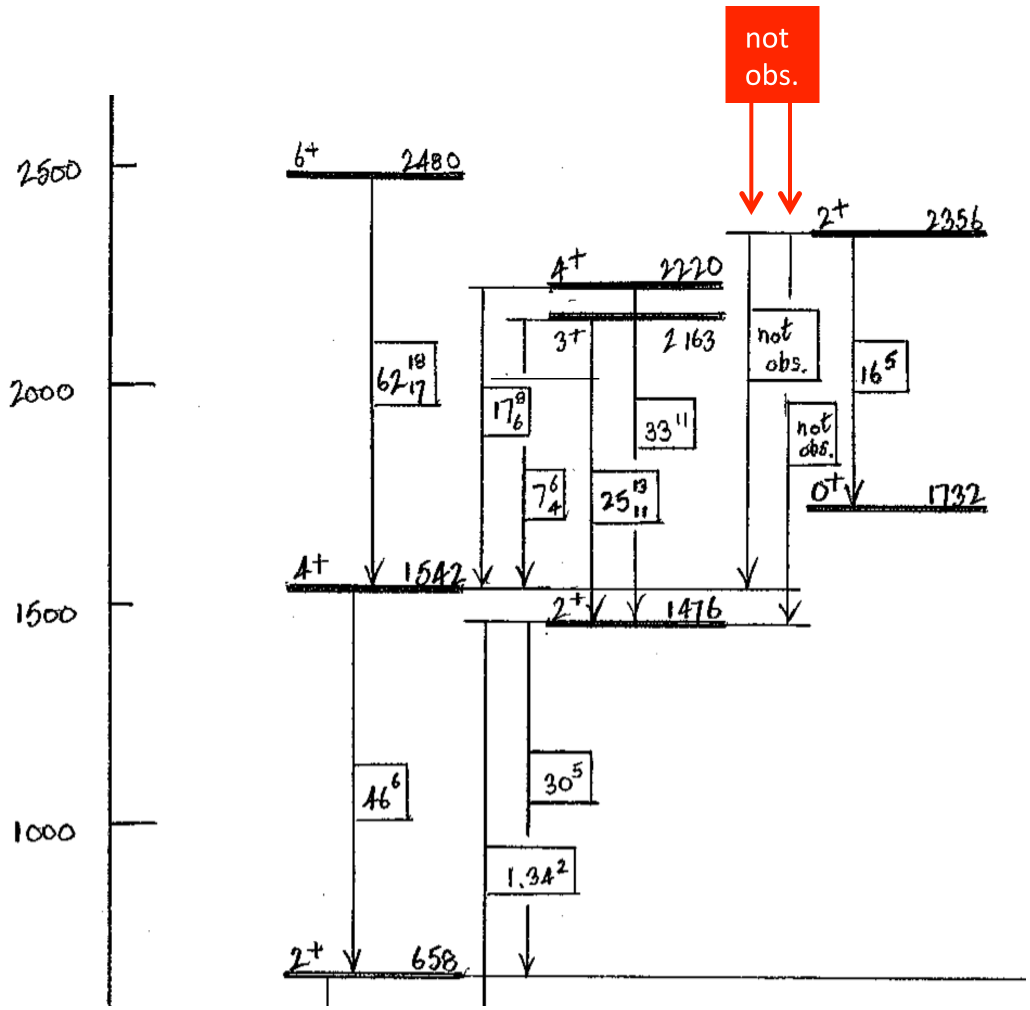


n = 0,1,2,3,4,5 !!

n = phonon No.

^{110}Cd: expected 3-phonon→ 2-phonon transitions not all observed

P.E. Garrett et al.,
Phys. Rev. C 86 044304 2012



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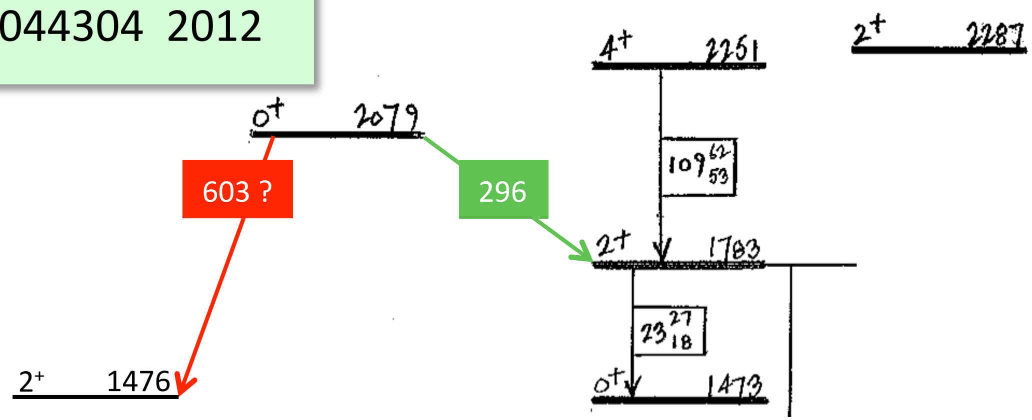
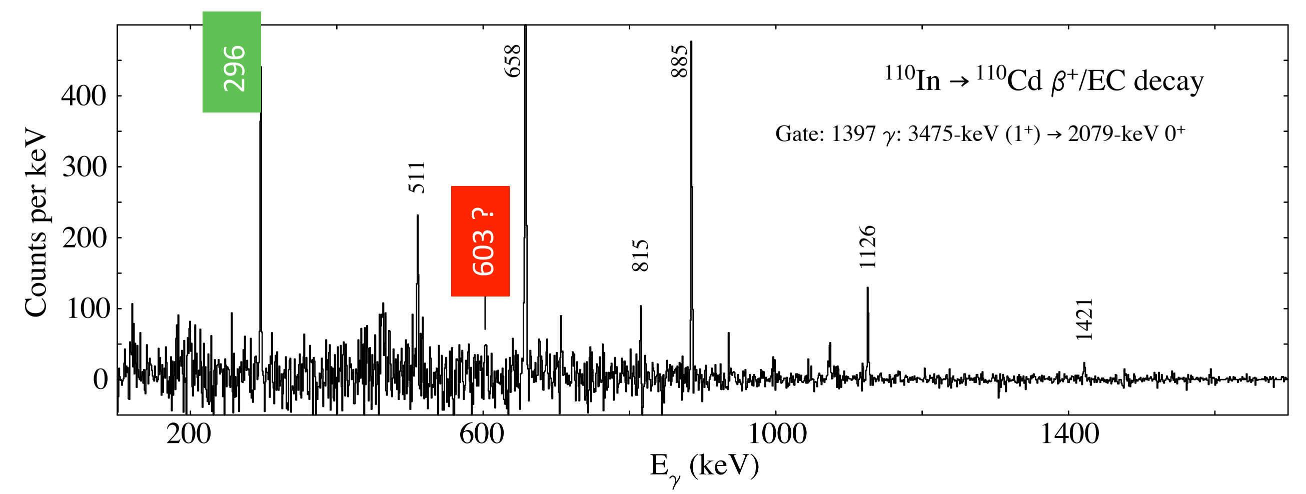
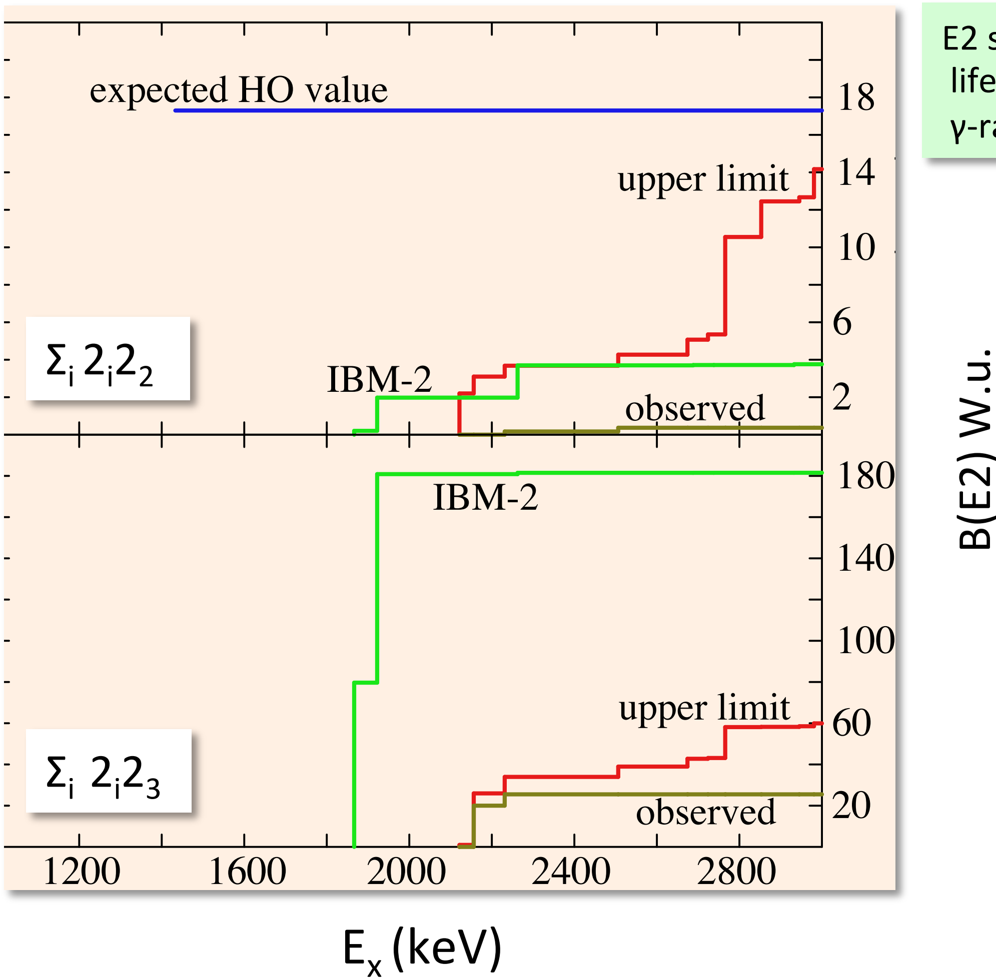
P.E. Garrett et al.,
Phys. Rev. C 86 044304 2012





Summed E2 strength (W.u.) to 2_2^+ (2-phonon) state and 2_3^+ (intruder) state in ^{112}Cd



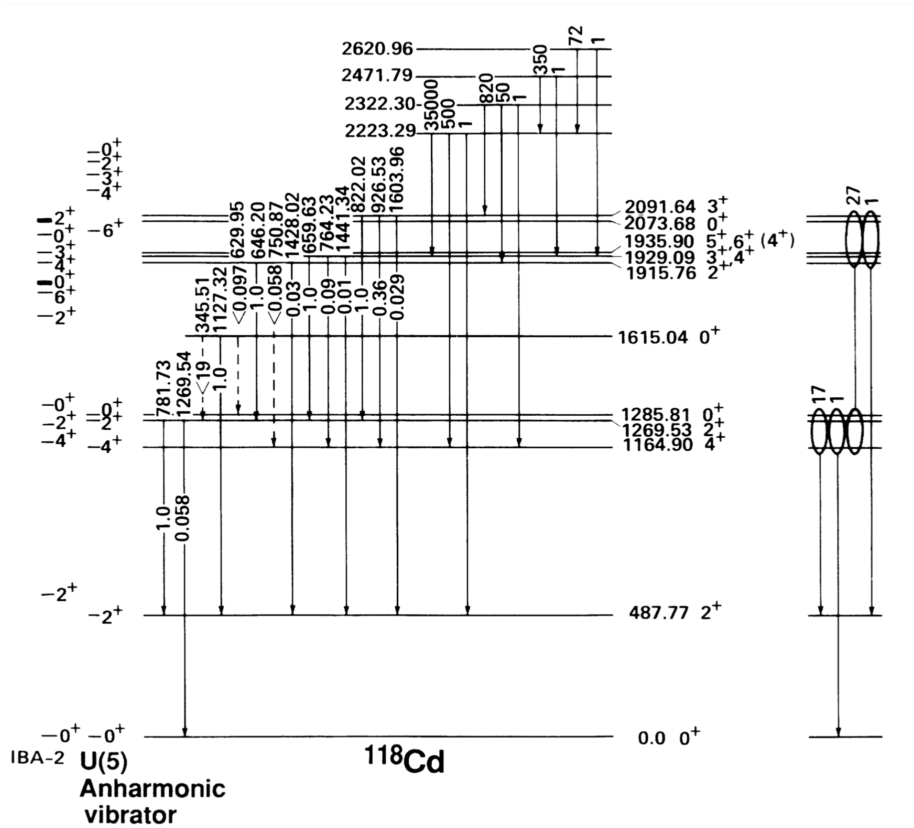
E2 strengths from:
lifetimes--(n,n'γ) Garrett PR C75 054310 2007
γ-ray branching—β decay In, Ag, Garrett (unpubl.)

Is ^{118}Cd a "Near-Harmonic Vibrational Nucleus"-- exhibiting multi-phonon states up to N = 5?

VOLUME 59, NUMBER 5 PHYSICAL REVIEW LETTERS 3 AUGUST 1987

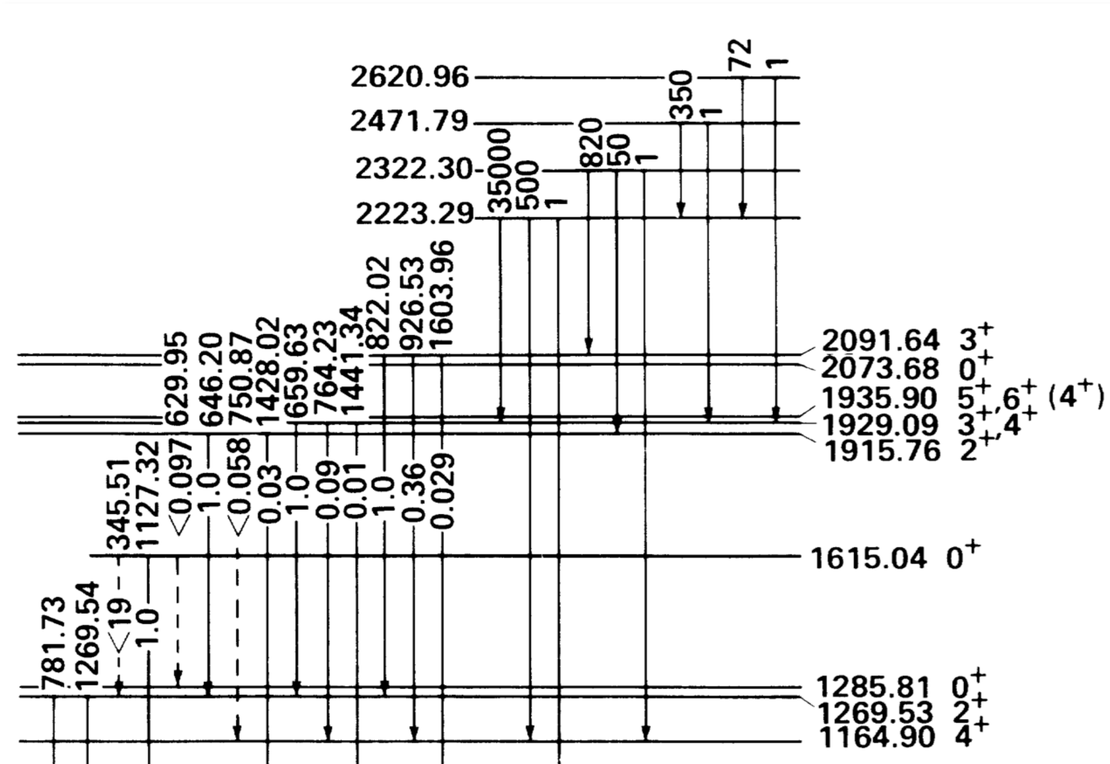
First Observation of a Near-Harmonic Vibrational Nucleus

A. Aprahamian, D.S. Brenner, R.F. Casten, and others



Finally, there is one other surprising feature of the empirical ^{118}Cd level scheme. Above the quintuplet, there are several levels whose relative decay favors population of quintuplet levels by orders of magnitude. For example, on the assumption of pure $E2$ multipolarities, Fig. 2 shows that the 2223- and 2322-keV levels populate levels of the quintuplet by low-energy γ rays whose relative $B(E2)$ values dominate the higher-energy decay transitions by factors of 35000 and 820. This gives rise to the speculation that these two levels may even contain amplitudes for a four-phonon structure. In addition, there are levels at still higher excitation energies which, in turn, show preferential decay to these levels. These low-energy decay routes are very unusual and defy any standard interpretation. (Note that, if they are not $E2$ but rather $M1$ with an E_γ^3 rather than E_γ^5 energy dependence, their dominance is numerically smaller but no less puzzling.)

Is ^{118}Cd a "Near-Harmonic Vibrational Nucleus"-- exhibiting multi-phonon states up to N = 5?



Errors in parity assignments:

2223.29 5^-

2471.79 5^-

Wang PR C67 064303 2003

Luo NP A874 32 2012

Systematics of neg. par. states:

1929.09 3^-

2322.30 4^-

Systematics of intruder states:

1615.04 0^+

1915.76 2^+

Lifetime of 1285.81 0_2^+ state yields:

$B(E2; 0_2^+ \rightarrow 2_1^+) = 5.3^{0.8}$ W.u.

(harm. vib. expect 66 W.u.)

Mach PRL 63 143 1989

Is ^{118}Cd a "Near-Harmonic Vibrational Nucleus"--exhibiting multi-phonon states up to N = 5?

Finally, there is one other surprising feature of the empirical ^{118}Cd level scheme. Above the quintuplet, there are several levels whose relative decay favors population of quintuplet levels by orders of magnitude. For example, on the assumption of pure $E2$ multipolarities, Fig. 2 shows that the 2223- and 2322-keV levels populate levels of the quintuplet by low-energy γ rays whose relative $B(E2)$ values dominate the higher-energy decay transitions by factors of 35000 and 820. This gives rise to the speculation that these two levels may even contain amplitudes for a four-phonon structure. In addition, there are levels at still higher excitation energies which, in turn, show preferential decay to these levels. These low-energy decay routes are very unusual and defy any standard interpretation. (Note that, if they are not $E2$ but rather $M1$ with an E_γ^3 rather than E_γ^5 energy dependence, their dominance is numerically smaller but no less puzzling.)

Aprahamian et al.
PRL 59 535 1987

"assumption of pure E2":
--erroneous, they are E1

"four-phonon structure":
--erroneous, they are negative-parity states

"very unusual and defy any standard interpretation":
--see above two remarks

ANSWER: NO!—CONCLUSIONS ARE BASED ON UNFOUNDED ASSUMPTIONS.

Universal rotor B(E2)'s

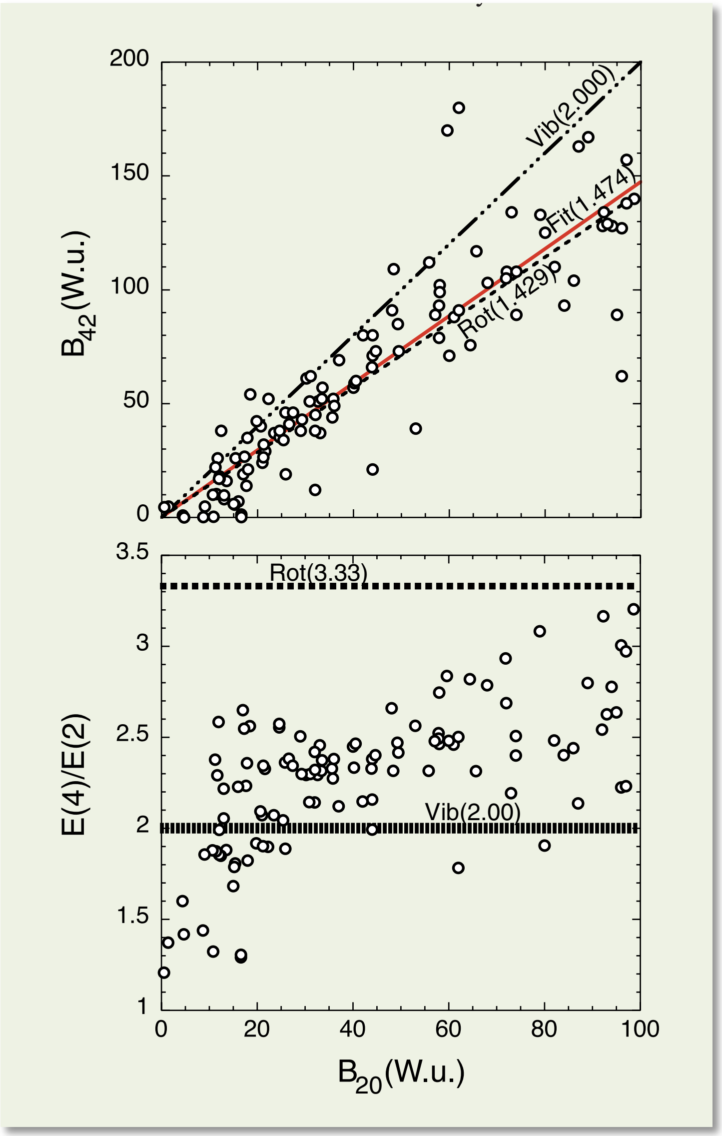


Figure from:
Thiamova, Rowe, and Wood
NP A780 112 2006

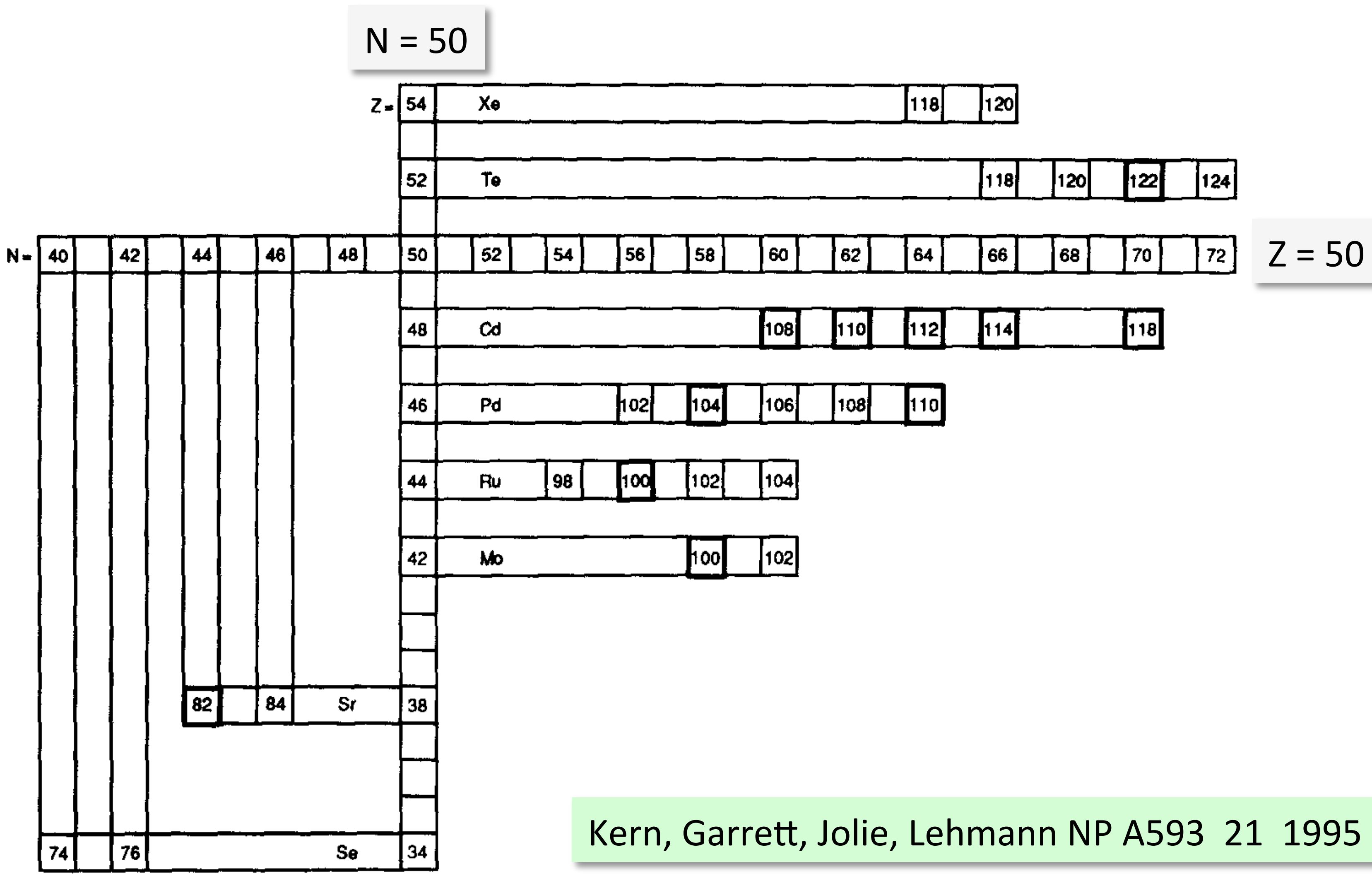
see also Heyde and Wood
Rev. Mod. Phys. 83 1467 2011

Best candidate nuclei for low-energy quadrupole vibrations

Isotope	R_4	$E(0_2^+)$	$E(2_2^+)$	$E(4_1^+)$	$\frac{B_{2_2 0_1}}{B_{2_2 2_1}}$	$\frac{B_{4_1 2_1}}{B_{2_1 0_1}}$	$\frac{B_{2_2 2_1}}{B_{2_1 0_1}}$	$\frac{B_{0_2 2_1}}{B_{2_1 0_1}}$
^{62}Ni	1.99	2048	2302	2336	0.045	–	–	–
^{80}Kr	2.33	1321	1256	1436	0.012	1.2^2	0.7^1	–
^{82}Sr	2.32	1311	1176	1328	≈ 0.006	2.3^5	–	–
^{98}Ru	2.14	1322	1414	1398	0.022	0.4^1	1.4^5	–
^{104}Pd	2.38	1334	1342	1324	0.055	1.4^2	0.6^1	0.4^1
^{106}Pd	2.40	1134	1128	1229	0.027	1.6^1	1.0^1	0.8^1
^{108}Pd	2.42	1053	931	1048	0.011	1.5^2	1.4^1	1.1^1
^{110}Pd	2.46	947	814	921	0.014	1.7^2	1.0^2	0.6^1
^{118}Te	1.99	957	1151	1206	>0.006	–	–	–
^{120}Te	2.07	1103	1201	1162	0.026	–	–	–
^{122}Te	2.09	1357	1257	1181	0.011	–	–	–

R&W
Table 1.1

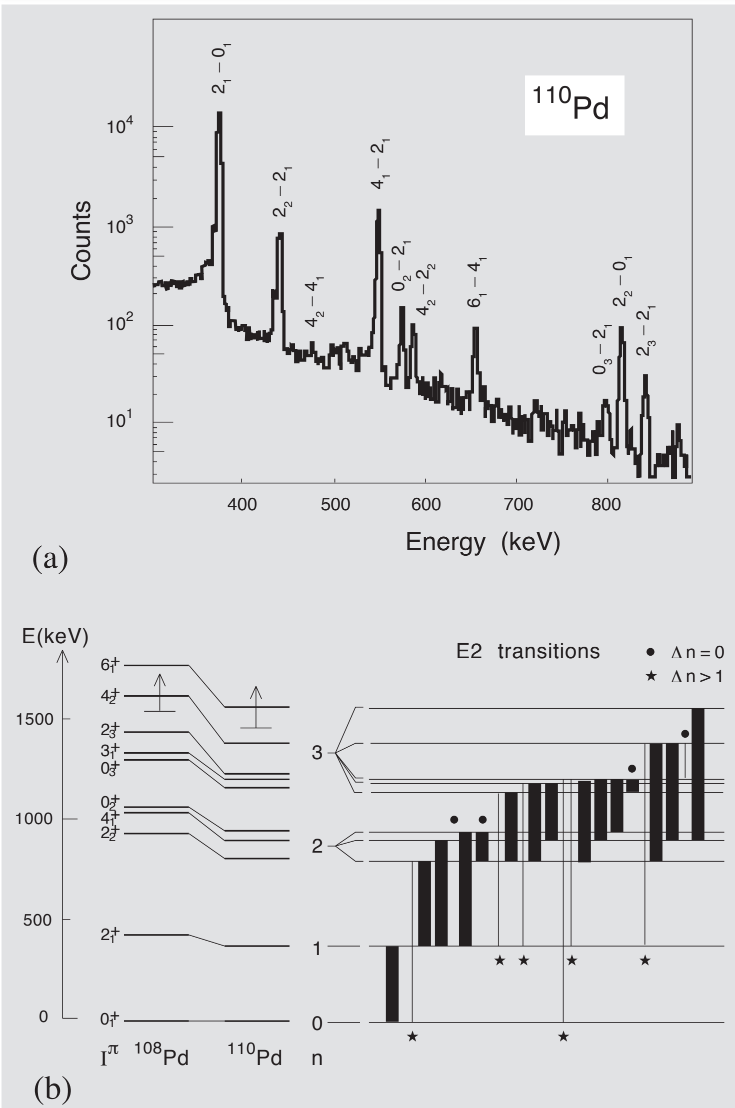
Results of search of Jean Kern et al. for U(5)



Kern, Garrett, Jolie, Lehmann NP A593 21 1995

Coulex view of a quadrupole vibrational nucleus?

Rowe and Wood Fig. 1.50



E2 properties of ^{106,108,110}Pd from multi-Coulex (Svensson) cf. harmonic quadrupole vibrator

	th.	^{106}Pd expt.	^{106}Pd $\frac{\text{expt.}}{\text{th.}}$	^{108}Pd expt.	^{108}Pd $\frac{\text{expt.}}{\text{th.}}$	^{110}Pd expt.	^{110}Pd $\frac{\text{expt.}}{\text{th.}}$
$b_{4_1 2_1}$	2	1.72	0.86	1.55	0.78	1.64	0.82
$b_{2_2 2_1}$	2	0.93	0.47	1.02	0.51	0.88	0.44
$b_{0_2 2_1}$	2	1.03	0.52	1.08	0.54	0.52	0.26 •
$b_{2_2 0_1}$	0	0.02		0.01		0.01	
$b_{6_1 4_1}$	3	2.16	0.72	2.05	0.68	1.97	0.66
$b_{4_2 4_1}$	1.43	0.56	0.39	0.61	0.43	0.58	0.41
$b_{4_2 2_2}$	1.57	0.85	0.54	1.11	0.71	0.62	0.39
$b_{3_1 4_1}$	0.86	0.14	0.16 •	–	–	0.23	0.27 •
$b_{3_1 2_2}$	2.14	0.41	0.19 •	0.52	0.24 •	0.46	0.21 •
$b_{2_3 2_2}$	0.57	0.25	0.44	0.11	0.19 •	0.26	0.46
$b_{2_3 0_2}$	1.4	0.93	0.66	1.54	1.10	1.41	1.01
$b_{2_3 4_1}$	1.03	0.13	0.12 •	1.12	1.09	1.06	1.03
$b_{0_3 2_2}$	3	0.34	0.11 •	0.12	0.04 •	0.95	0.36
$b_{4_2 2_1}$	0	2×10^{-4}		4×10^{-3}		3×10^{-3}	
$b_{3_1 2_1}$	0	0.01		0.01		7×10^{-3}	
$b_{2_3 0_1}$	0	3×10^{-3}		2×10^{-3}		6×10^{-3}	
$b_{2_3 2_1}$	0	0.01		0.03		0.02	
$b_{0_3 2_1}$	0	0.06		0.01		0.04	
$Q(2_1)$	0	-0.72		-0.83		-0.87	
$Q(2_2)$	0	$+0.52$		$+0.73$		$+0.70$	
$Q(4_1)$	0	-1.02		-0.82		-1.6	

$b_{42} = B_{42}/B_{20}$ etc.

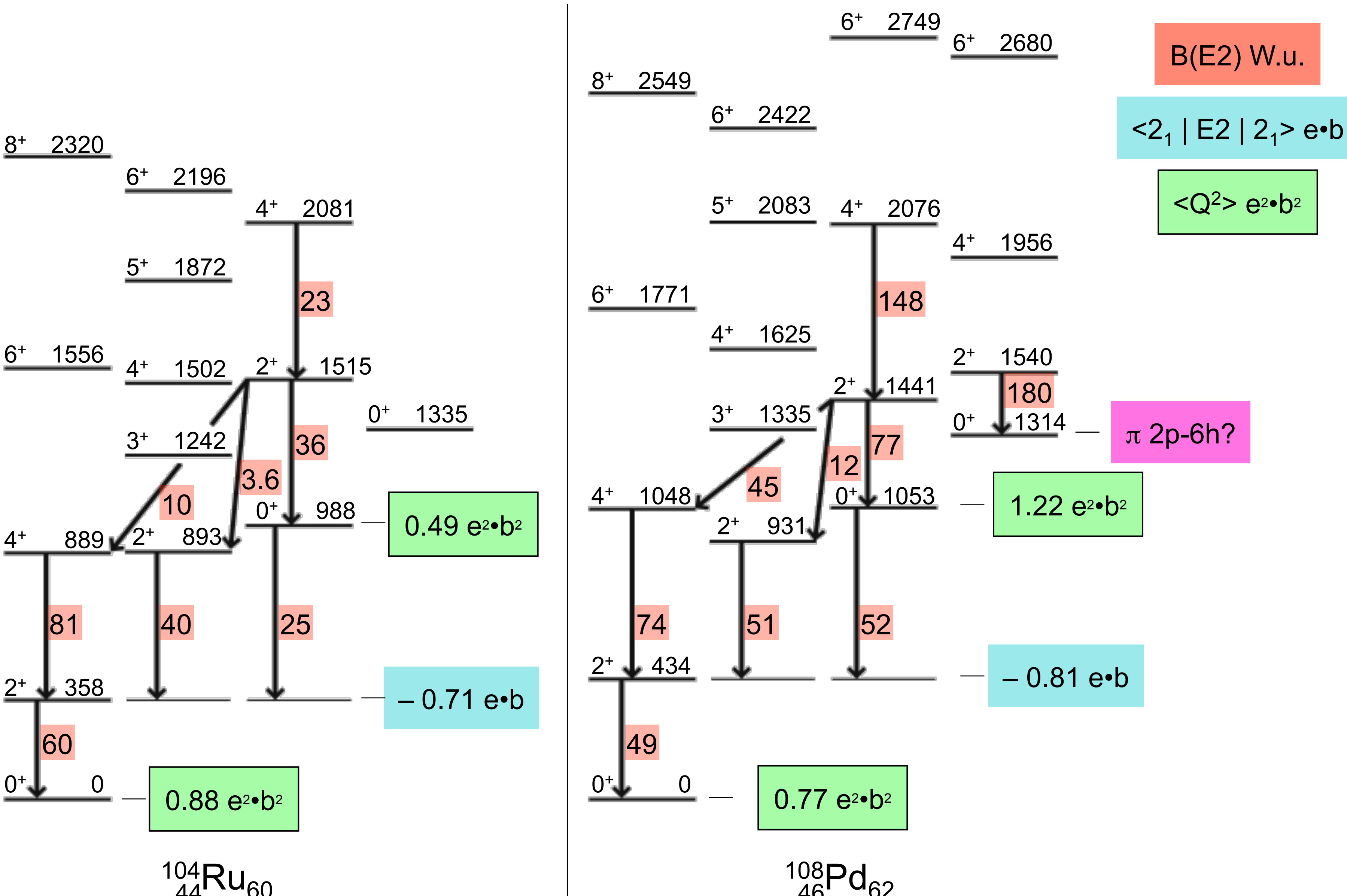
• Disagrees strongly with theory

Overall, there is a deficiency of E2 strength

Large quadruple moments indicate deformation

R&W Table 2.2

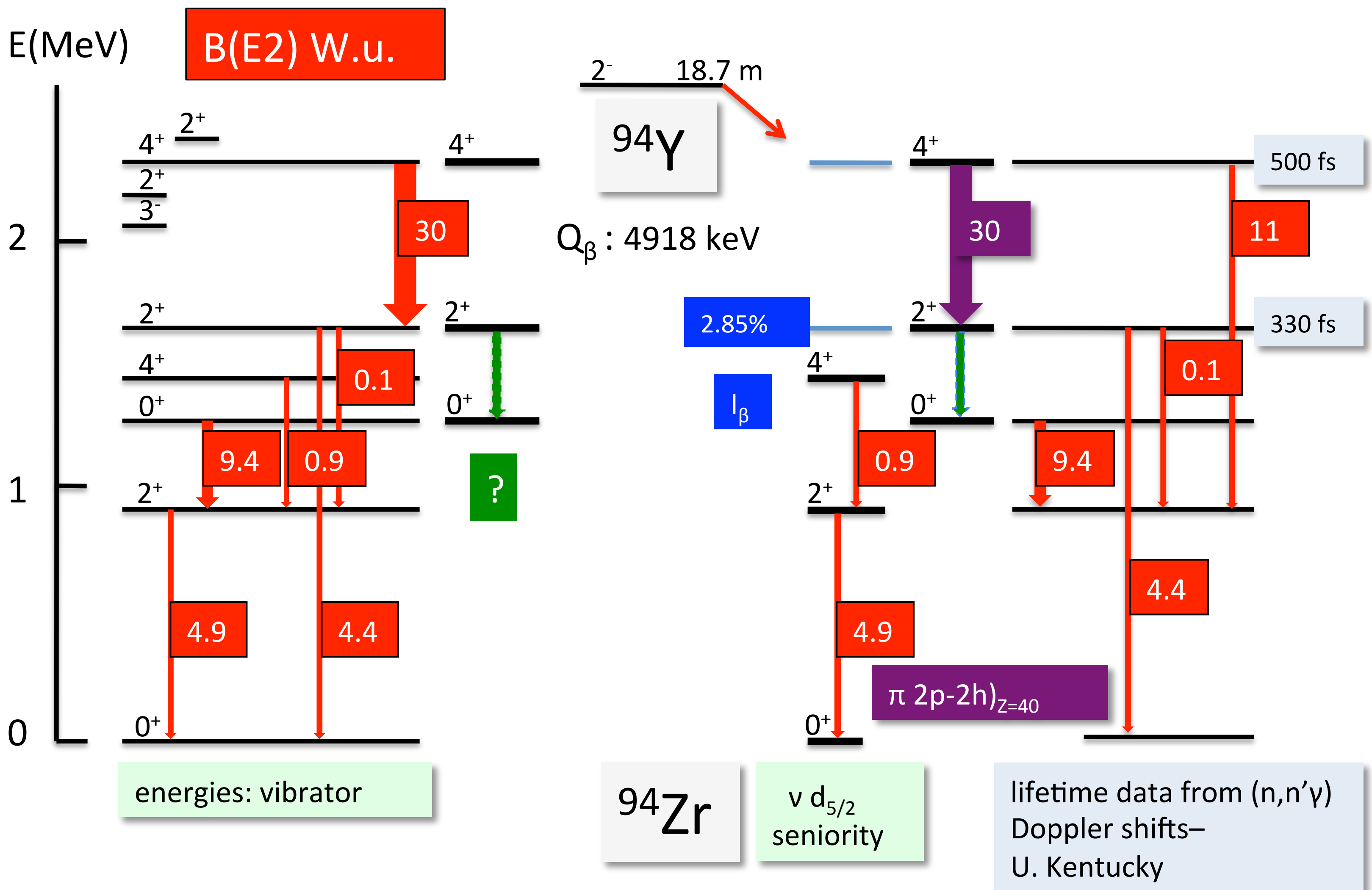
Collective states in ^{104}Ru and ^{108}Pd from multi-Coulex



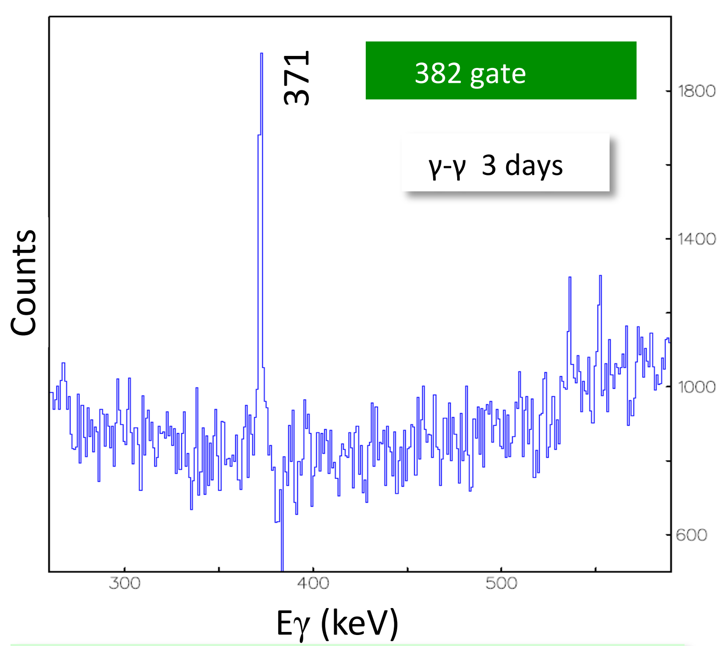
J. Srebrny et al.
Nucl. Phys. **A766**, 25(2006)

L. E. Svensson et al.
Nucl. Phys. **A584**, 547(1995)

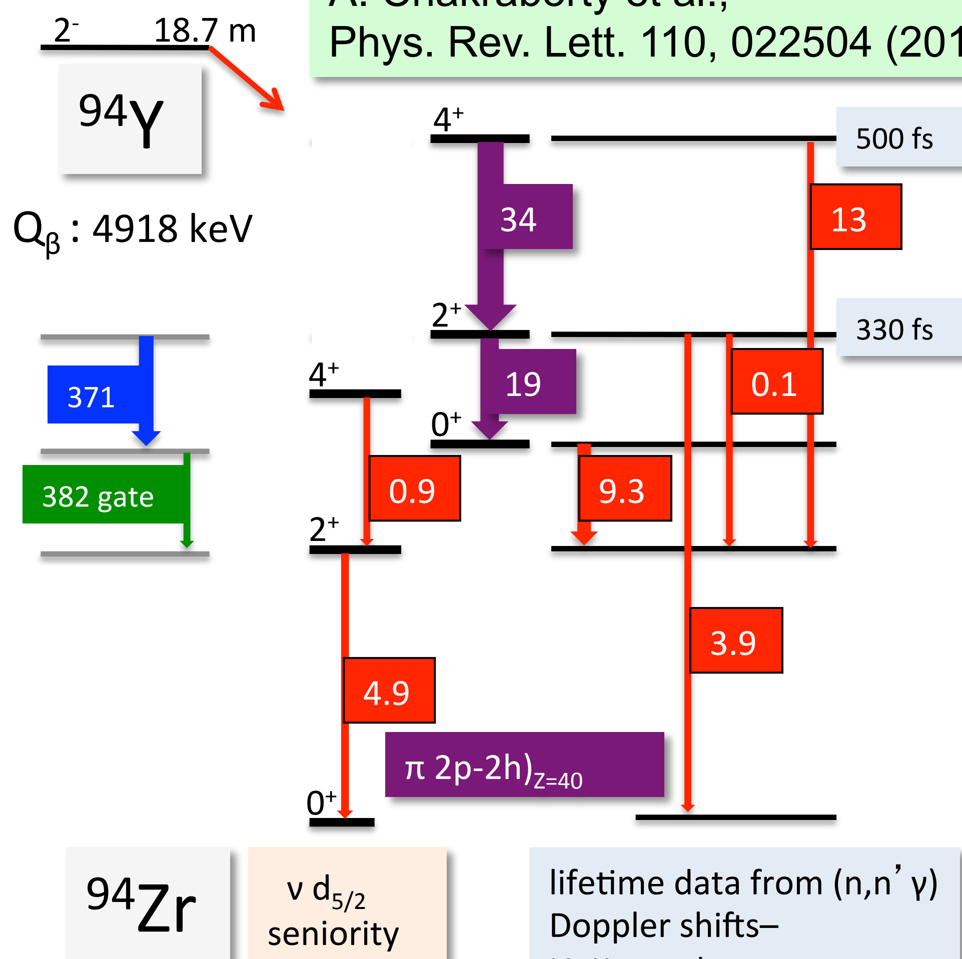
^{94}Zr from two structural perspectives: vibrator OR coexisting seniority and deformed structures



^{94}Zr from two structural perspectives: vibrator OR coexisting seniority and deformed structures



A. Chakraborty et al.,
Phys. Rev. Lett. 110, 022504 (2013)



gamma branches from the 2^+ 2p-2h state to:

0_1^+ state 58

2_1^+ state 42 (δ = 0.02: 0.04% E2)

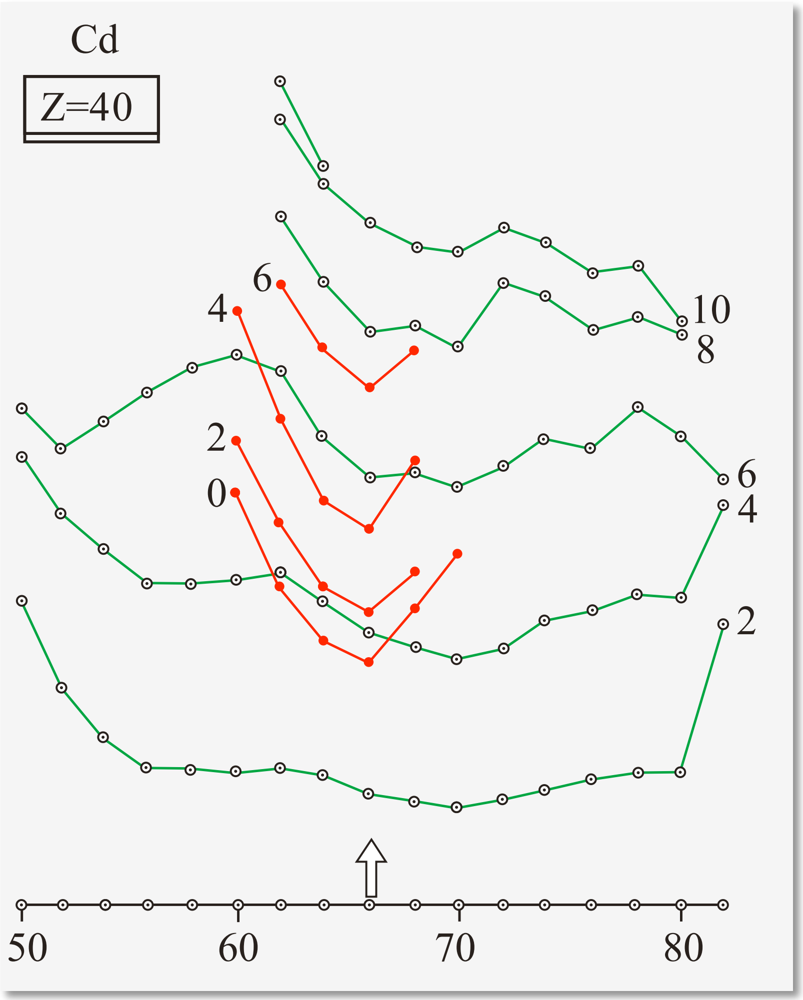
0^+ 2p-2h state 0.15

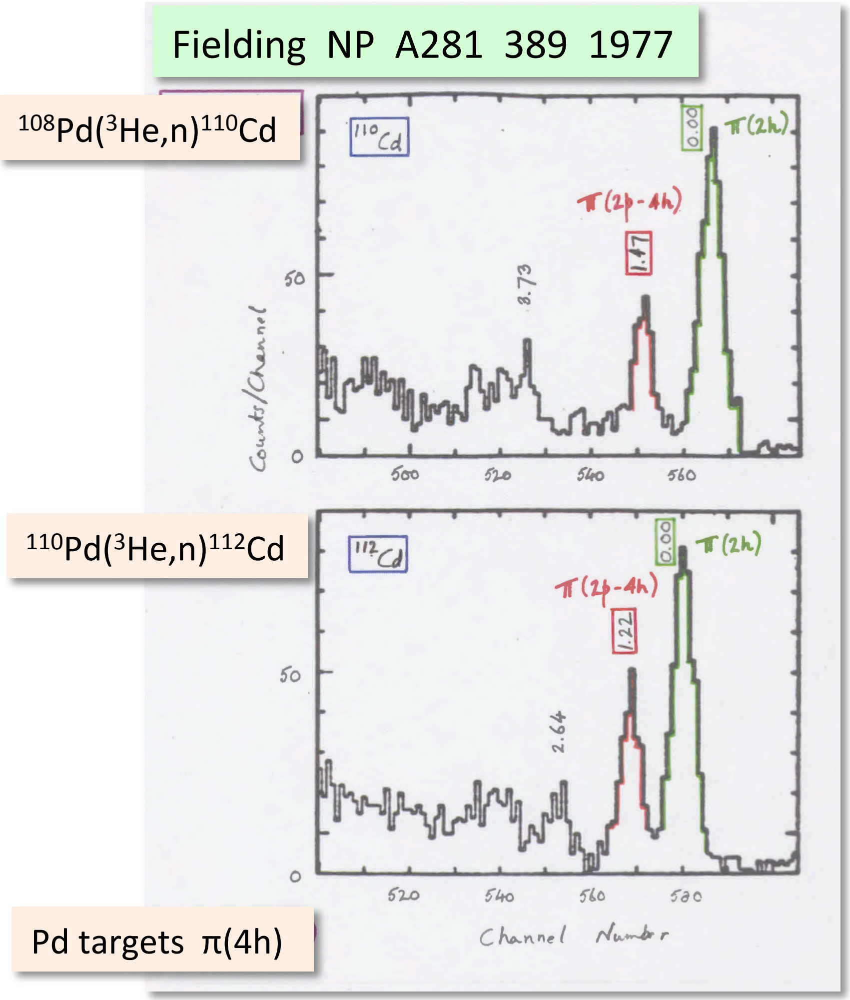
E. Elhami et al., PR C78, 054303 (2008)

8Pi expt.: S.W. Yates et al.,S1286 (5 days)

lifetime data from (n,n' γ) Doppler shifts–
U. Kentucky

Shape coexistence in the Cd isotopes

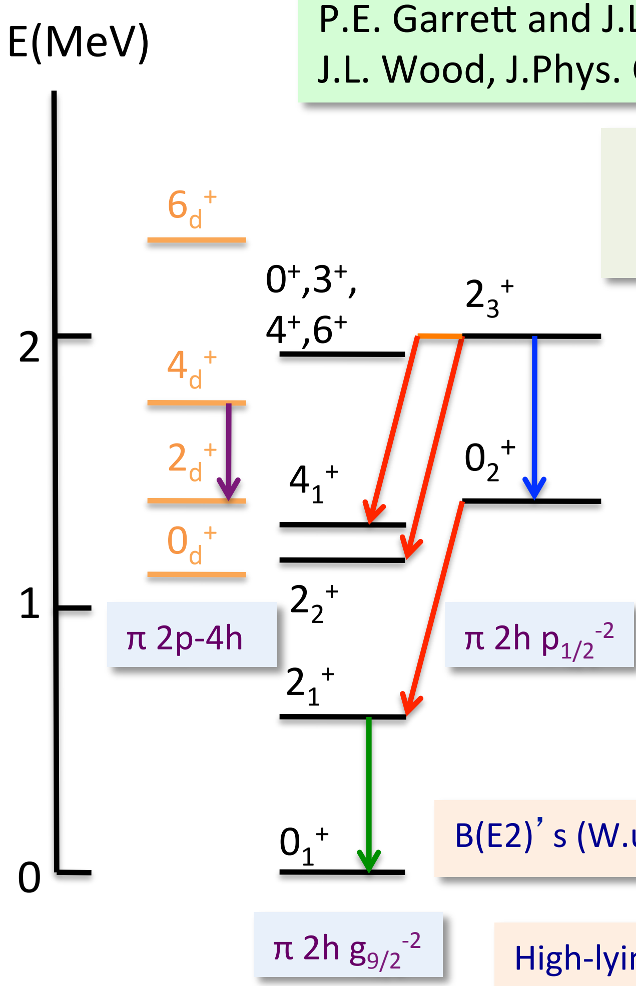




Demise of quadrupole vibrations in $^{110-116}$Cd:

low-energy 0^+ states are shell and subshell excitations

P.E. Garrett and J.L. Wood, J.Phys. G37, 064028 (2010)
J.L. Wood, J.Phys. Conf. Ser. 403, 012011 (2012)



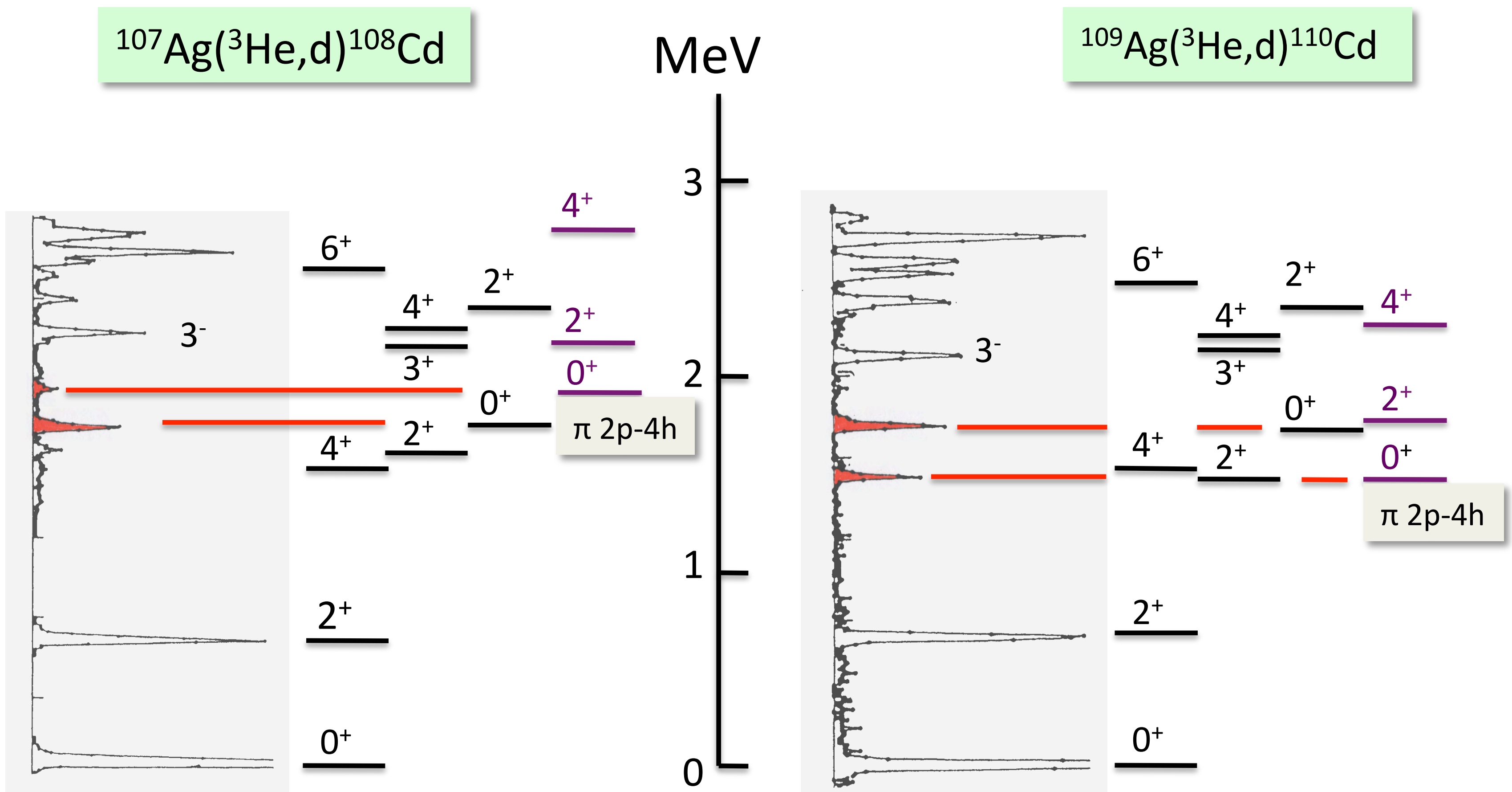
trans.	^{110}Cd	^{112}Cd	^{114}Cd	^{116}Cd	harm. vib.
$4_d 2_d$	115^{35}		119^{12}		
$2_3 0_2$	24^{2}	27^{8}	17^{5}	35^{10}	42
$2_3 4_1$	< 5	< 0.4	< 0.3	< 7	31
$2_3 2_2$	< 0.7^{6}	< 2	2.8^{4}	2.0^{6}	17
$0_2 2_1$	< 7.9	0.0099^{44}	0.0026^{4}	0.55^{4}	60
$2_1 0_1$	27	30	31	34	30 (norm)

B(E2)'s (W.u.) from lifetime measurements @ Univ. of Kentucky using (n,n'γ).

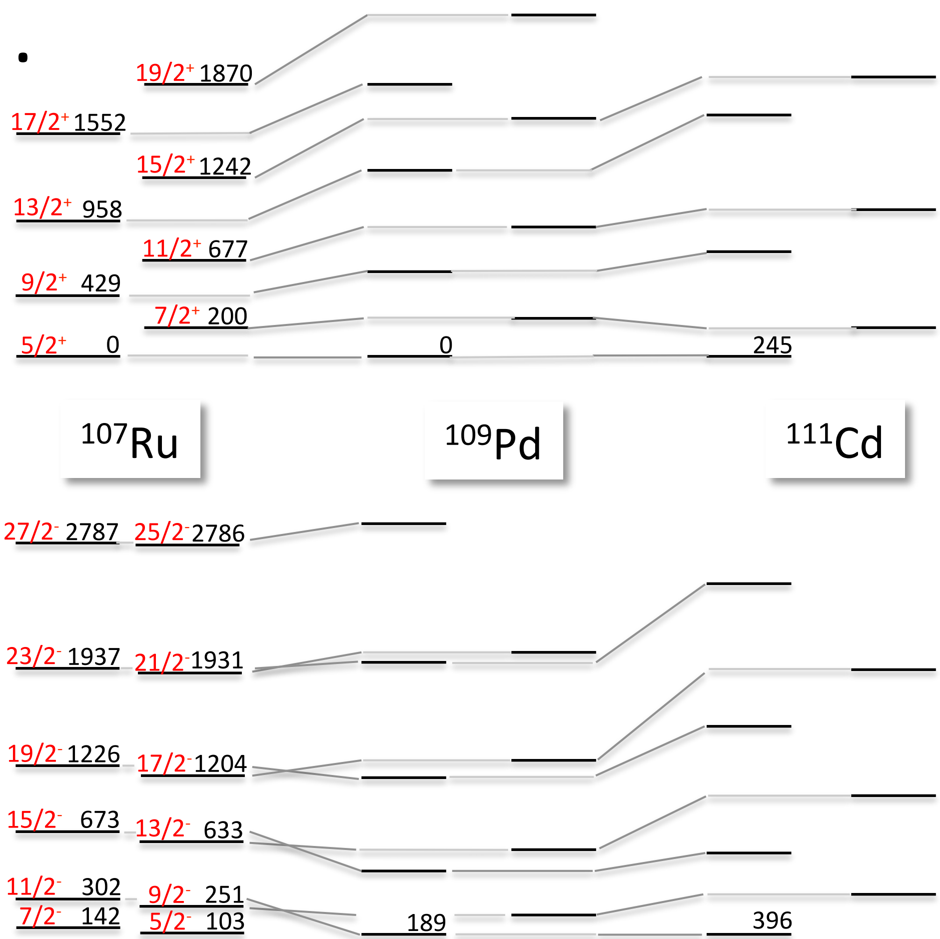
High-lying, low-energy transitions are difficult to observe: used 8Pi array @ TRIUMF-ISAC and ultrahigh statistics β-decay scheme studies.

Population of 0^+ states in $^{108,110}Cd$ by one-proton stripping reactions

R.L. Auble et al., Phys. Rev. C6 2223 (1972)



Rotational bands in ^{111}Cd, ^{109}Pd, ^{107}Ru



Symmetric rotor interpretation of transitional nuclei—
Simms et al.
NP A347 205 1980

The spectroscopy of mixing in the Cd isotopes (schematic): ^{114}Cd unmixed energies

^{114}Cd : unmixed energies

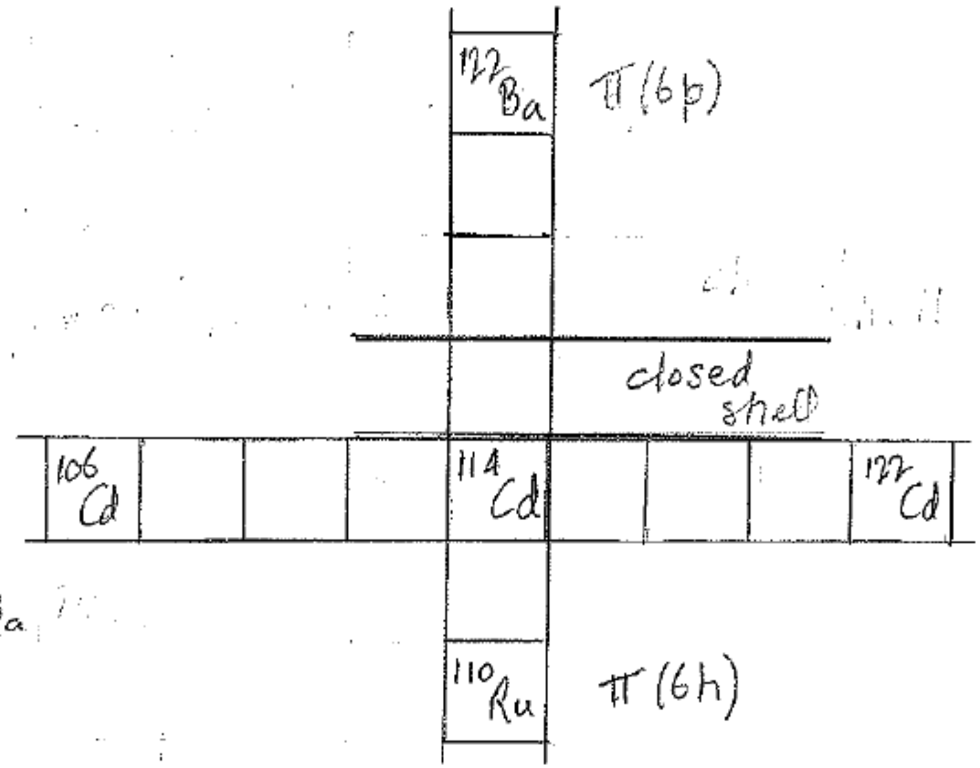
^{114}Cd $\pi(2h)$

	avg.	^{106}Cd	^{122}Cd
2_1	601	633	569
4_1	1412	1494	1329
2_2	1543	1717	1368

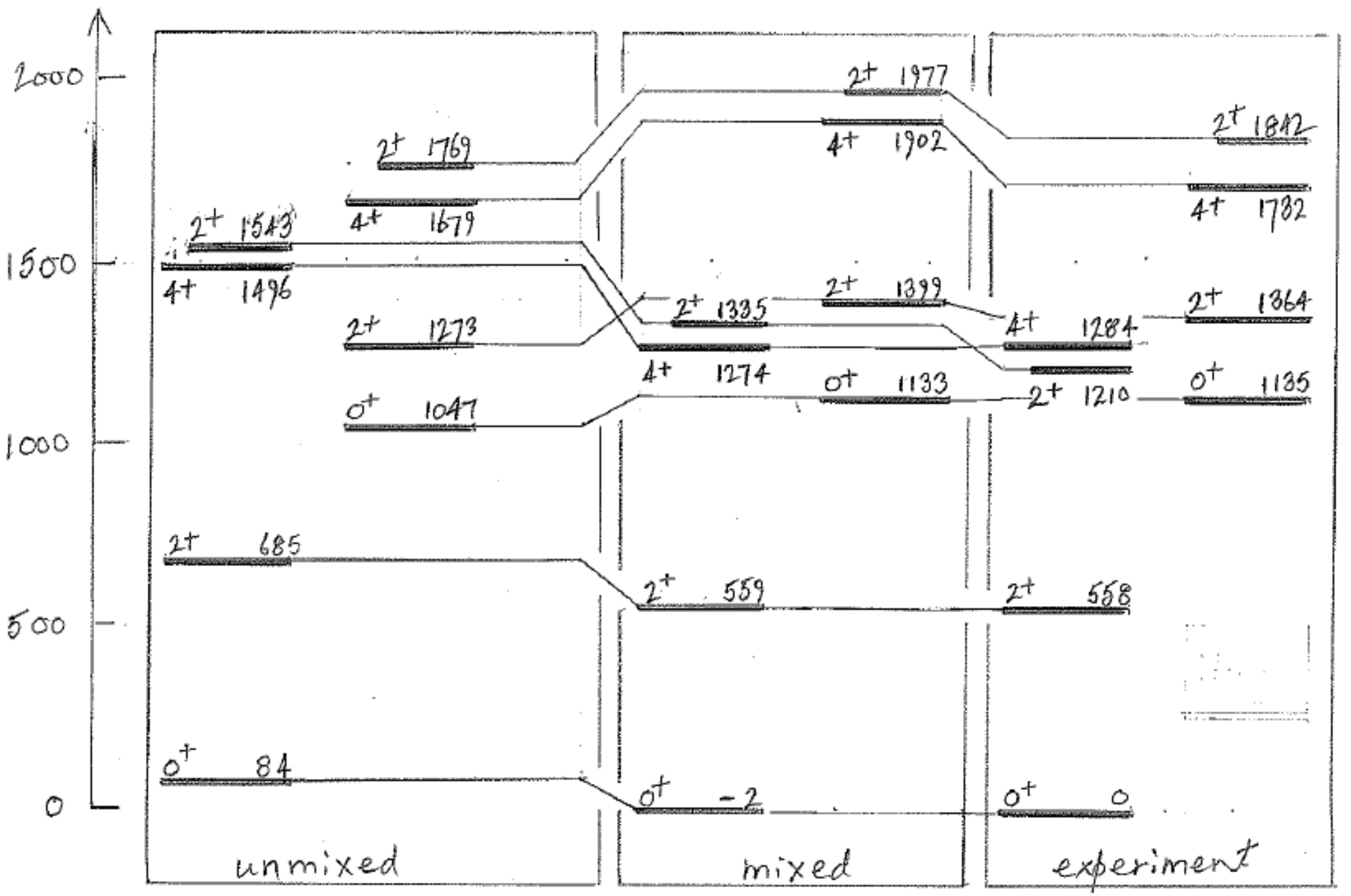
$\pi(2p\text{-}4h)$

	"avg."	^{110}Ru	^{122}Ba
2_1	226	241	196
4_1	632	663	569
2_2	722	613	940

$$\text{"avg."} \equiv \frac{2}{3} \times {}^{110}\text{Ru} + \frac{1}{3} \times {}^{122}\text{Ba}$$



The spectroscopy of mixing in the Cd isotopes (schematic): ^{114}Cd energies



V_J ~ 300 keV, all J

The spectroscopy of mixing in the Cd isotopes (schematic): ρ^2 (E0) values in ^{114}Cd

J_i	α_J	β_J	$\rho^2_{J\to J}(E0) = 228\,\alpha_J^2\beta_J^2$	$\rho^2_{J\to J}(E0)$ expt.
0_1	0.9613	0.2765	16	~~16±1~~ 19
2_1	0.9220	0.3872	29	~~36±5~~ 43
4_1	0.8038	0.5948	52	~~67±10~~ 89
2_2	0.8218	0.5698	50	~~95±19~~ 122

The spectroscopy of mixing in the Cd isotopes (schematic): M(E2) and B(E2) values in ^{114}Cd

★ B(E2) properties:

Grodzins' rule: $[E(2_1^+)\ \mathrm{keV}][B(E2; 0_1^+ \to 2_1^+)\ e^2 \cdot b^2] \frac{A}{Z^2} \approx 16.0$

$$M_{20} = \sqrt{B(E2; 0_1^+ \to 2_1^+)}\ e \cdot b$$

$E(2_1^+)^a$ 601 keV $\Rightarrow$ $M_{20}^a = 0.73\ e \cdot b$

$E(2_1^+)^b$ 226 keV $\Rightarrow$ $M_{20}^b = 1.20\ e \cdot b$

	calc.	expt.
$M_{2_1 0_1} = \alpha_0 \alpha_2 M_{20}^a + \beta_0 \beta_2 M_{20}^b$	$= 0.775$	0.714^{21}
$M_{4_1 2_1} = (\alpha_2 \alpha_4 M_{20}^a + \beta_2 \beta_4 M_{20}^b)(1.604)$	$= 1.311$	1.35^{4}
$M_{0_2 2_1} = -\alpha_2 \beta_0 M_{20}^a + \alpha_0 \beta_2 M_{20}^b$	$= 0.261$	0.300^{+7}_{-9}
α_2 -1.5% $\Rightarrow$	0.300	
‡ $M_{4_2 2_3} = (\beta_2 \beta_4 M_{20}^a + \alpha_2 \alpha_4 M_{20}^b)(1.604)$	$= 1.696$	1.85^{+10}_{-6}
$M_{2_3 0_2} = \beta_0 \beta_2 M_{20}^a + \alpha_0 \alpha_2 M_{20}^b$	$= 1.142$	0.51^{3}

α_2 -1.5% $\Rightarrow$ $\rho^2(E0)_{2\to2}$: $29 \to 33$ (36^5)

$B(E2; 2_1 \to 0_1) = \frac{M_{2_1 0_1}^2}{5}\ e^2 \cdot b^2 \times 302.3\ \frac{\mathrm{W.u.}}{e^2 \cdot b^2}$: 36 cf. 33^2 Raman

$B(E2; 0_2 \to 2_1) = M_{0_2 2_1}^2$: 21 cf. 27.2^{14}; α_2 -1.5% $\to$ 27

$B(E2; 4_1 \to 2_1) = \frac{M_{4_1 2_1}^2}{9}$: 58 cf. 61^4

$B(E2; 4_2 \to 2_3) = \frac{M_{4_2 2_3}^2}{9}$: 97 cf. 115^{+13}_{-8}

$B(E2; 2_3 \to 0_2) = M_{2_3 0_2}^2 / 5$: 79 cf. 65^9 / but see Fahlander

$M_{2_1 2_1} = (\alpha_2^2 M_{20}^a + \beta_2^2 M_{20}^b)(-1.195) = -0.957\ e \cdot b$ cf. $-0.36^{+1}_{-3}\ e \cdot b$

need 2_1–2_2 mixing

‡ α_2 decreases by 1.5% for $\lambda_2^{(1)}$: $559 \to 547$ keV